Dissecting Business Cycles

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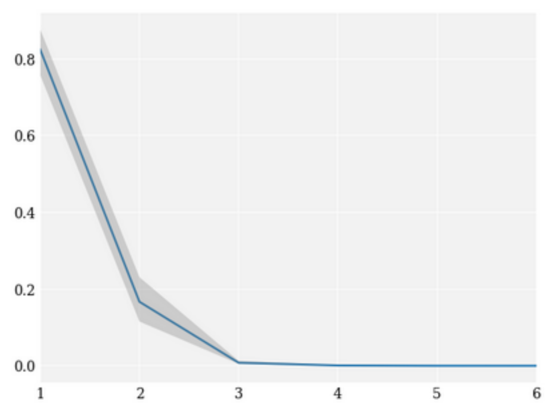
Abstract

This paper proposes decomposing macroeconomic fluctuations into three components based on their contributions at different frequencies. The first component explains business cycle fluctuations but has no long-run effect; the second has both; the third only long-run effects. The first two components jointly explain 99% of business cycle fluctuations and deliver comovements consistent with a structural interpretation as demand and supply shocks, respectively. The third component's presence biases full-information estimation of business cycle models, distorting conclusions about the sources of business cycles. Estimating a model to match only business cycle shocks resolves this issue and yields more plausible parameter estimates.

Figure 1: Scree Plot



Eigenvalues for a spectral matrix of GDP at business cycle frequency band. Horizontal axis: component number. Vertical axis: eigenvalue.

Are business cycles driven by a single "main shock"? A recent strand of literature assumes so (e.g., Angeletos et al. (2020)), and draws several surprising empirical and theoretical conclusions about the likely causes of cycles. But, this assumption is testable and it is rejected by the data. Formal tests, summarized by

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the scree plot in Figure 1, show that business cycle fluctuations are best described as having a two-factor structure: procedures that impose a single source of business cycles risk conflating the effects of two distinct structural shocks.

In this paper, I propose a new approach to identify the main business cycle shocks, plural. The objective is similar to that of the literature cited above: to learn about the causes of business cycles, without imposing ex-ante structure on the underlying fundamental sources of fluctuations. Productivity, expectations, financial shocks and more are all plausible candidates that my procedure could identify. My goal, however, is to do so without imposing the restrictive single-shock assumption that has played an important role in this recent literature. I show that relaxing this assumption substantially changes the conclusions I draw about the sources of business cycles, and that an "old fashioned" dichotomy of supply and demand shocks does a very good job overall of accounting for macroeconomic covariances at the business cycle frequency.

Motivated by figure 1, this paper proposes to separate the multiple potential sources of business cycles based on their effects at different frequencies (persistence). Intuitively, this approach resembles the early literature on long-run restrictions (e.g., Blanchard and Quah (1989), Gali (1999)). An important difference, however, is that I decompose the data into three components. The underlying data generating process (DGP) assumes three orthogonal shocks: two business cycle shocks and one residual, labeled as a non-business cycle shock. Building on the work of Uhlig (2003) and Angeletos et al. (2020) (ACD), the study identifies two business cycle shocks that together explain the maximum business cycle volatility in real per capita GDP. I restrict first of these shocks from explaining long-run GDP fluctuations, so that it only drives business cycle fluctuations. I label it a 'short-run business cycle' shock. The other business cycle shock remains unrestricted, allowing it to explain GDP fluctuations at both long-run and business cycle frequencies. I label it a 'long-run business cycle' shock. This results in a third component or a residual shock that is allowed to explain only long-run fluctuations and no fluctuations at business cycle frequencies and hence classified as a non-business cycle shock. Thus, the framework accounts for two types of long-run fluctuations: one that drives business cycles and one that does not. The empirical findings reveal the significant presence of both types of long-run shocks, which has important implications for the full-information estimation of Dynamic Stochastic General Equilibrium (DSGE) models.

The two shocks permitted to explain business cycle fluctuations collectively account for approximately 99% of business cycle volatility in GDP. This result aligns with the two-factor structure suggested in Figure 1. Both shocks independently contribute to business cycle comovements, explaining a significant portion of volatility in GDP, consumption, investment, hours worked, and unemployment, justifying their classification as business cycle shocks. I analyze the impulse response functions (IRFs) to the identified shocks in order to understand if they have a straightforward structural interpretation. I label the 'short-run business cycle' shock, subject to the restriction of not explaining long-run fluctuations, as a 'BC-demand' shock, because it leads to significant procyclical fluctuations in real GDP, inflation, and federal funds rates. Meanwhile, the unrestricted 'long-run business cycle' shock is labeled a 'BC-supply' shock because it causes countercyclical fluctuations in inflation and federal funds rate, along with long-run impulse responses for output, consumption, investment, TFP, and labor productivity, contributing approximately 51% of long-run volatility across these variables.

The second part of the paper studies the implications of having two types of long-run shocks. I argue that cross-frequency restrictions within DSGE models, coupled with the significant presence of long-run non-business cycle shocks, can lead to biased parameter estimates, thereby impacting the normative implications of these models. While the two identified business cycle shocks account for approximately 99% of the business cycle volatility in GDP, they explain only about 51% of the long-run volatility in both GDP and productivity measures. This discrepancy suggests that a significant portion of long-run fluctuations are not associated with business cycles. I label the shocks driving these fluctuations, 'long-run non-business cycle' (LR-NBC) shocks. The presence of these non-business cycle fluctuations in the data-generating process challenges the parameter estimation of medium-scale DSGE models when using a full information setting.

I show that the presence of 'long-run non-business cycle' shocks implies that full-information estimation procedures will systematically underestimate business cycle volatility in DSGE models. This argument rests on two key steps. First, the maximization of the log-likelihood in the frequency domain can be reformulated as minimizing a sum of weighted ratios over all frequencies, where each ratio consists of data-implied volatility weights in the numerator and model-implied volatility corresponding to each frequency in the denominator. The presence of 'long-run non-business cycle' shocks in the data places greater weight on long-run frequencies. Consequently, parameter estimation adjusts to generate more model-implied volatility at long-run frequencies to achieve this minimization. Second, because DSGE models focused on explaining business cycle fluctuations have inherent frequency restrictions, parameter adjustments that increase long-run model-implied volatility result in reduced volatility at business cycle frequencies. This introduces a downward bias in model-implied business cycle volatility to accommodate the higher long-run volatility. I theoretically argue that the log-likelihood function reaches its maximum at the true parameters only when the volatility of non-business cycle shocks is zero, proving that their presence induces a downward bias.

To emphasize the normative and policy implications of models estimated under full information with non-business cycle fluctuations, I next utilize the benchmark medium-scale DSGE model proposed by Smets and Wouters (2007) (SW07). First, the identification strategy is applied to the seven-variable VAR in SW07, yielding empirical results similar to the identified business cycle shocks from the benchmark VAR of ACD. Originally estimated using Bayesian likelihood estimation, the SW07 model is re-estimated using impulse response matching methodology akin to Christiano et al. (2005) with the identified business cycle shocks. Re-estimating the model through IRF matching requires selecting two structural shocks out of the seven in SW07, based on the identified conditional correlations of business cycle shocks. The SW07 model has only two structural shocks that can replicate the comovements characteristic of the identified business cycle shocks. This allows for further structural interpretation: the "BC-supply" shock aligns with a total factor productivity (TFP) shock, while the "BC-demand" shock corresponds to a risk premia shock. This approach not only provides a further structural interpretation of the identified 'BC-demand' shock but also reduces the number of parameters by eliminating five of the seven shocks, addressing the structural misspecification concerns raised by Chari et al. (2009) (CKM09).

CKM09 argue that New Keynesian models are not ready for quarter-to-quarter policy analysis. They contend that the drive to fit macroeconomic data leads to the inclusion of non-structural shocks and questionable mechanisms, resulting in models with a large number of parameters that are inconsistent with microeconomic evidence. CKM09 advocate for simpler models with fewer, micro-founded parameters that are more consistent with underlying economic theory. Re-estimating the model through IRF matching with the identified business cycle shocks offers a solution by disciplining the model to capture the relevant moments in the data while simultaneously reducing the number of parameters.

The impulse response matching reveals a key policy tradeoff for monetary authorities, as it identifies a negative comovement between inflation and the output gap, in contrast to the positive comovement observed in the SW07 model when estimated under full information. This underscores how sensitive policy recommendations are to the estimation methodology, particularly due to non-business cycle fluctuations. Additionally, it resolves the issue of backward price indexation, which CKM09 argue is inconsistent with micro data[1]. The impulse response matching results in a price indexation parameter close to zero, aligning with micro data, in contrast to the Bayesian likelihood estimate, further validating the identification strategy.

Related Literature: This paper makes contributions to two distinct strands within the macroeconomic literature on business cycles. The first strand, grounded in the Structural Vector Autoregression (SVAR) framework, centers on assessing the relative impact of aggregate supply and demand shocks on business cycle fluctuations. Existing research in this strand, such as Blanchard and Quah (1989), Shapiro and Watson (1998), & Angeletos et al. (2020), collectively advocates for demand shocks as the primary drivers of business cycles, challenging the traditional view from Real Business Cycle (RBC) models that place technology (TFP) shocks at the forefront. Empirical studies, including Gali (1999), Basu et al. (2006), Barsky and Sims (2011), Barsky et al. (2014), Neville et al. (2014a), Kurmann and Sims (2021), Basu et al. (2006), and Kurmann and Sims (2021), Benhima and Poilly (2021), contest the prevailing notion of TFP fluctuations as the key business cycle driver, revealing zero or negative associations between hours and productivity for short-run and long-run technology shocks. Most literature has assumed a single type of long-run productivity shock, whereas I consider two types of long-run supply shocks: one influencing business cycles and the other not. This departure relaxes the assumption of a singular long-run supply shock and aligns with SVAR evidence presented by Beaudry and Portier (2006), Chahrour and Jurado (2018), and Chahrour et al. (2023).

Two contemporaneous studies, Forni et al. (2024) and Granese (2024), seek to resurrect the traditional Blanchard and Quah (1989) view of supply and demand driven business cycles. These studies are centered around the concern that the smaller-scale VAR of ACD might be susceptible to informational deficiency problems. To address this, both studies employ a large-dimensional Structural Dynamic Factor model using a dataset of 114 quarterly U.S. time series. In contrast, this paper proposes an alternative identification strategy within the ACD benchmark VAR, applying restrictions across different frequencies including the separation of long-run frequency fluctuations into long-run business cycle shocks and long-run non-business cycle shocks. This distinction is crucial for counterfactual policy analysis, as the identified conditional correlations of the two business cycle shocks allow for a more accurate estimation of DSGE model parameters, which might otherwise be biased by non-business cycle fluctuations. This identification strategy is implemented within the seven-variable VAR framework of SW07 to estimate the model parameters, with results on the role of supply and demand shocks remaining robust even in a smaller VAR than that of ACD.

The second strand of literature to which this paper contributes is the literature on the identification of

[1] Bils and Klenow (2004), Golosov and Robert E. Lucas (2007), Midrigan (2010), Nakamura and Steinsson (2008).

macroeconomic equations through structural shocks. Notable works in this domain include Rotemberg and Woodford (1997) and Christiano et al. (2005), which estimate DSGE models by matching impulse response functions. More recently, Barnichon and Mesters (2020) introduced a new approach involving regressions in impulse response space, and Lewis and Mertens (2022) presented an improved approach. This paper extends this literature by advocating for the use of conditional variation in identified business cycle shocks to discipline structural model parameters, as opposed to the Bayesian likelihood approach using unconditional moments due to the presence of non-business cycle fluctuations.

The paper is organized as follows. Section I outlines the data and methodology employed to dissect business cycle fluctuations and identify business-cycle (BC) supply and demand shocks. In Section II, the primary empirical findings are presented. Building on the evidence of long-run non-business cycle shocks from the previous section, Section III formally discusses the issue of biased estimation in a full information setting. Moving on to Section IV, I estimate the parameters of the medium-scale DSGE model of SW07 required to match the identified dynamic causal effects of business cycle shocks. This section also undertakes a comparative analysis of the model, estimated using conditional moments, and the Bayesian likelihood estimation of the original estimated model. Finally, Section V concludes.

I Data and Method

The main analysis uses the benchmark VAR from ACD with quarterly data on ten key U.S. macroeconomic variables: unemployment, real per capita GDP, hours worked, investment, consumption, labor productivity, utilization-adjusted TFP, labor share, inflation (GDP deflator), and the federal funds rate. The sample spans from 1955:I to 2019:IV

The Vector Autoregression (VAR) model employed in this study takes the form:

$$A(L)Y_t = \mu_t, \tag{1}$$

where Y_t is a vector of n macroeconomic variables under examination, $A(L)$ is a matrix polynomial represented by the sum of $A_\tau L^\tau$, with $A(0) = A_0 = I$, and l is the number of lags included in the VAR. The vector of residuals, μ_t, has covariance $E(\mu_t \mu_t') = \Sigma$ for a positive definite matrix Σ. The VAR is estimated using Bayesian methods and a Minnesota prior. The baseline specification has 2 lags, as suggested by standard Bayesian criteria.

The method assumes a linear relationship between the residuals μ_t and a set of mutually orthogonal shocks ε_t, given by $\mu_t = C\varepsilon_t$, where C is an invertible $n \times n$ matrix.

The shocks ε_t are assumed to be i.i.d. with identity covariance matrix I. To identify them, C is decomposed as $C = \tilde{C}Q$, where $\tilde{C}$ is the Cholesky factor of the residuals' covariance matrix and Q is an orthonormal matrix. This yields

$$\varepsilon_t = C^{-1}\mu_t = Q'\tilde{C}^{-1}\mu_t,$$

with each column of Q corresponding to a distinct shock.

However, simply satisfying $QQ' = I$ and $\tilde{C}\tilde{C}' = \Sigma$ does not suffice for identifying the underlying shocks. To do so, we impose additional restrictions on Q based on the requirement that it contains the maximal share of all the information in the data about the volatility of a specific variable in a specific frequency band. This approach differs from typical SVAR exercises in the literature, which often rely on exclusion or sign restrictions motivated by specific theoretical assumptions. Here, the structural interpretation of the orthogonal shocks is informed by their conditional correlations, allowing us to investigate the drivers of business cycles without imposing ex-ante assumptions on the fundamental sources of fluctuations. This method can potentially identify various plausible drivers—such as productivity, expectations, and financial shocks—based solely on the comovements of macroeconomic variables in the VAR.

The Wold representation of the VAR model is given by the following equation:

$$Y_t = B(L)\mu_t, \tag{2}$$

where $B(L)$ is an infinite matrix polynomial, and μ_t represents the residuals. We then substitute $\mu_t = \tilde{C}Q\varepsilon_t$, where $\tilde{C}$ is the Cholesky decomposition of the VAR residuals covariance matrix, and Q is an orthonormal matrix, leading to the following representation:

$$Y_t = D(L)Q\varepsilon_t = \Theta(L)\varepsilon_t, \tag{3}$$

where $D(L)$ and $\Theta(L)$ are infinite matrix polynomials, with $D_\tau \equiv B_\tau\tilde{C}$ and $\Theta_\tau \mathrm{V} D_\tau Q$ for all $\tau \in 0, 1, 2, \ldots$. The sequence $\{\Theta_\tau\}_{\tau=0}^{\infty}$ represents the impulse response functions (IRFs) of the variables to the identified shocks.

As mentioned, the interpretation of the structural shocks is based on the dynamic responses of the variables to the respective shock. By considering the (i, j) element of the matrix Θ_τ , one may identify the effect of the jth shock on the ith variable at horizon τ.

A Identification Strategy

The identification strategy used in this paper builds upon the “max-share" approaches first introduced in the literature by Faust (1998) and Uhlig (2003). These approaches have been subsequently adapted and expanded upon by several other authors, including Barsky and Sims (2011), Kurmann and Otrok (2013), Neville et al. (2014b), Angeletos et al. (2020), Kurmann and Sims (2021) and Charhour et al. (2021) among others. The implementation of this strategy is in the frequency domain, similar to that used by Angeletos et al. (2020) (ACD) where the goal is to identify a reduced-form shock that explains the maximum volatility of a targeted variable in a specific frequency band.

In contrast to the ACD approach, this paper allows for two distinct shocks as potential drivers of business cycle fluctuations. This standpoint is underpinned by the scree plot illustrated in Figure 1, where the x-axis is limited in accordance with the maximum number of eigenvalues of a 10-variable VAR. The observed trend in the eigenvalues reveals a convergence to zero from the third eigenvalue onward, while the first two principal components retain significant values. Econometric tests suggest that two factors are necessary to adequately explain business cycle fluctuations. In contrast, the ACD approach primarily emphasizes the first eigenvalue, which accounts for 80% of business cycle volatility in output, as the main business cycle shock. Drawing from this empirical evidence, the argument advanced in this paper considers two orthogonal business cycle shocks to effectively account for the fluctuations in real GDP per capita across business cycles.

The key novel contribution of this paper lies in the dual employment of targeting and constraining methodologies to identify the two orthogonal business cycle shocks and better explain the volatility of specific variables over a certain frequency band while constraining them to fluctuations of another variable in the same or a different frequency domain.

To illustrate, I entertain a linear data generating process (DGP), where the vector of structural shocks ε_t can be decomposed into two distinct categories: business cycle shocks and non-business cycle shocks.

$$\epsilon_t' = [\ \underbrace{\epsilon_{B,t}^{sr} \qquad \epsilon_{B,t}^{lr}}_{\text{Business cycle shocks}}\ \underbrace{\epsilon_{NB,t}^{long-run} \qquad \epsilon_t^{residual}}_{\text{Non-Business Cycle shocks}}\ ]$$

The business cycle shocks are represented as $\epsilon_{B,t}^{sr}$ and $\epsilon_{B,t}^{lr}$, while the non-business cycle shocks are represented as $\epsilon_{NB,t}^{long-run}$ and $\epsilon_t^{residual}$. The subscript 'B' in $\epsilon_{B,t}^{sr}$ and $\epsilon_{B,t}^{lr}$ denotes that these orthogonal shocks are identified by maximizing their contribution to the volatility of the targeted variable, real GDP per capita, over the business cycle frequency band, or frequencies pertaining to a time period of 6-32 quarters.

The superscript 'sr' in $\epsilon_{B,t}^{sr}$ denotes a restriction of the shock to explain long-run volatility of real GDP per capita at long-run frequencies, i.e. frequency bands pertaining to time periods of 20-100 years. In other words, $\epsilon_{B,t}^{sr}$ is identified by simultaneously targeting the same variable and frequency bands as $\epsilon_{B,t}^{lr}$, but with the additional restriction of explaining long-run volatility of real GDP per capita. The results are robust if the restriction is applied to long-run fluctuations of consumption, TFP or labor productivity instead of GDP.

On the other hand, non-business cycle shocks are further classified into two subcategories: long-run shocks and residual shocks. The long-run shocks, identified as $\epsilon_{NB,t}^{long-run}$, are orthogonal to the business cycle shocks and lead to persistent changes in real GDP per capita or fluctuations pertaining to frequency bands with time periods above 20 years. These shocks explain long-run fluctuations of GDP but not the business cycle fluctuations. On the other hand, the residual shocks, identified as $\epsilon_t^{residual}$, are orthogonal to both the business cycle shocks and the long-run non-Business cycle shocks. These residual shocks capture all other non-business cycle shocks that are not captured by the other two categories.

The objective of this paper is to partially identify the business cycle shocks, and this further classification of ϵ_t allows for a more detailed examination of the results. This classification of structural shocks is useful for highlighting the results of our analysis in comparison to existing literature, such as the works of Blanchard and Quah (1989) and Angeletos et al. (2020). The inclusion of non-business cycle shocks introduces model misspecification, impacting both the SVAR approach and the local identification analysis of linearized DSGE models in full-information contexts. Sections III & IV of the paper elucidates this phenomenon using a SW07 framework.

In the Wold representation from the previous subsection, the variable Y_t can be represented as:

$$Y_t = D(L)Q\varepsilon_t$$

where ε_t is a white noise process and Q is an orthonormal matrix. The spectral density of a variable y_j in Y_t

in the frequency band $[\underline{f}, \bar{f}]$ can be represented by $\mathcal{D}(y_j, \underline{f}, \bar{f})$:

$$\mathcal{D}(y_j, \underline{f}, \bar{f}) = \int_{\underline{f}}^{\bar{f}} \left(\overline{D^j \left(e^{-if}\right)} D^j \left(e^{-if}\right) \right) df$$

where the sequence $\{D_\tau\}_{\tau=0}^{\infty}$ represents the Cholesky transformation of the VAR residuals, and D_τ^j represents the jth row of the matrix D_τ.

To identify a shock $\epsilon_{1,t}$, we need to find the column of the orthonormal matrix Q that represents the shock and explains the maximum volatility of y_j in the frequency band $[\underline{f}, \bar{f}]$. This can be represented as:

$$q_1 \equiv \arg\max_{q} \int_{\underline{f}}^{\bar{f}} \left(\overline{D^j \left(e^{-if}\right) q} D^j \left(e^{-if}\right) q \right) df \tag{4}$$

$$\equiv \arg\max_{q} q^{'} \mathcal{D}(y_j, \underline{f}, \bar{f}) q \tag{5}$$

$$\text{s.t. } q' q = 1, \tag{6}$$

Similarly, if we need to identify a shock $\epsilon_{2,t}$ that explains the maximum volatility of y_j in the frequency band $[\underline{f}, \bar{f}]$, but not the volatility of y_k in the frequency band $[\underline{\omega}, \bar{\omega}]$. This can be represented as:

$$q_2 \equiv \arg\max_{q} \int_{\underline{f}}^{\bar{f}} \left(\overline{D^j \left(e^{-if}\right) q} D^j \left(e^{-if}\right) q \right) df - \int_{\underline{\omega}}^{\bar{\omega}} \left(\overline{D^k \left(e^{-i\omega}\right) q} D^k \left(e^{-i\omega}\right) q \right) d\omega \tag{7}$$

$$\equiv \arg\max_{q} q^{'} \mathcal{D}(y_i, \underline{f}, \bar{f}) q - q^{'} \mathcal{D}(y_k, \underline{\omega}, \bar{\omega}) q \tag{8}$$

$$\text{s.t. } q' q = 1, \tag{9}$$

Building on this, the objective is to identify two orthogonal shocks: $q_{B,t}^{sr}$ and $q_{B,t}^{lr}$. These shocks simultaneously should explain the volatility of real GDP per capita at business cycle frequency, but restricting $q_{B,t}^{sr}$ from explaining the long-run volatility of GDP. The objective function is as follows:

$$q_B^{sr}, q_B^{lr} \equiv \arg\max_{q_B^{lr}, q_B^{sr}} q_B^{lr\prime} \mathcal{D}\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_B^{lr} + q_B^{sr\prime} \left(\mathcal{D}\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) - \mathcal{D}\left(GDP, \frac{2\pi}{400}, \frac{2\pi}{80}\right) \right) q_B^{sr} \tag{10}$$

$$\text{s.t. } q_B^{lr\prime} q_B^{lr} = 1, q_B^{sr\prime} q_B^{sr} = 1, q_B^{lr\prime} q_B^{sr} = 0 \tag{11}$$

To examine business cycle fluctuations, the framework follows Stock and Watson (1999) where the business cycle frequency band is defined as $\left[\frac{2\pi}{32}, \frac{2\pi}{6}\right]$, while the long-run frequency band is specified as $\left[\frac{2\pi}{400}, \frac{2\pi}{80}\right]$. The upper bound of the long-run frequency band is based on the findings of ACD, while the lower bound of $\frac{2\pi}{400}$ (100 years or 400 quarters) instead of ≈ 0 is chosen to avoid any potential non-stationarity so that the identification procedure does not rely on imposing stationarity in the estimation stage.

Additionally, the optimization procedure is subject to the constraint $q_B^{lr\prime} q_B^{sr} = 0$, ensuring that the long-run and short-run frequency bands are orthonormal. This means that the inner product of the two vectors is zero and each vector has a unit length. The resulting shocks, q_B^{lr} and q_B^{sr}, are interpreted as supply and demand shocks, respectively, based on their respective impulse response functions.

The problem of identification, expressed in equation 10, can be represented as,

$$\max_{X \in \mathbb{R}^{n \times p}} \mathcal{F}(X), \quad \text{s.t. } X^\top X = I_p \tag{12}$$

where,

$$\mathcal{F}(X) = \sum_{i=1}^{k} \mathbf{x}_i^T \mathbf{A}_i \mathbf{x}_i \tag{13}$$

A maximization of the sum of quadratic forms generated by different matrices. The objective is to find the set of orthonormal elements in $\mathbb{R}^n$ (where $k \leqslant n$) that maximizes the functional $\sum_{i=1}^{k} \mathbf{x}_i^T \mathbf{A}\mathbf{x}_i$, subject to the constraint $X^\top X = I_k$.

This problem is distinct from a standard principal component analysis (PCA), where the objective is to find a system of k orthonormal elements in $\mathbb{R}^n$ (where $k \leqslant n$) that maximize the functional $\sum_{i=1}^{k} \mathbf{x}_i^T \mathbf{A}\mathbf{x}_i$. In PCA, it is well known that the solution to this optimization problem is given by the k largest eigenvalues of matrix $\mathbf{A}$ and their corresponding orthonormal eigenvectors. These eigenvectors, or principal components, represent the most informative directions in the data and capture the maximum amount of variation in the data.

While in equation 12 the objective is to find the maximum of sum of quadratic forms generated by different matrices. In most cases it cannot be the sum of the largest eigenvalues of corresponding matrices, because the eigenvectors corresponding to the maximal eigenvalues of $\mathbf{A}_i$'s are usually not pairwise orthogonal.

In Bolla and Ziermann (1998), the authors prove the existence and uniqueness of solutions for optimization problems of this nature. The solution is determined using an adaptive feasible Barzilai-Borwein-like (AFBB) algorithm. The global convergence of this algorithm is demonstrated in Jiang and Dai (2014), through the use of an adaptive nonmonotone line search.

II Empirical Results

This section presents the main empirical findings and discusses a few tentative lessons for theory.

Figure 2 illustrates the impulse response functions (IRFs) of all variables to the $q_{B,t}^{lr}$ shock. This shock induces co-movement among key business cycle variables and plays a substantial role in explaining business cycle volatility, accounting for over 50% of the volatility in real per capita GDP. Additionally, the TFP impulse response to the same shock exhibits remarkable persistence, explaining approximately 54% of its long-run volatility. Furthermore, the shock is responsible for significant fluctuations in consumption and investment, both at business-cycle frequencies and in the long run.

In the context of conditional correlations involving real GDP, TFP, inflation, and interest rates, the shock has been identified as a long-run TFP shock within a new Keynesian model. In accordance with a common assumption in the business cycle literature, long-run TFP changes are considered exogenous. With the presence of nominal rigidities, a positive TFP shock leads to a decrease in inflation due to a decline in marginal cost, coupled with a decrease in interest rates following a standard inflation targeting monetary rule.

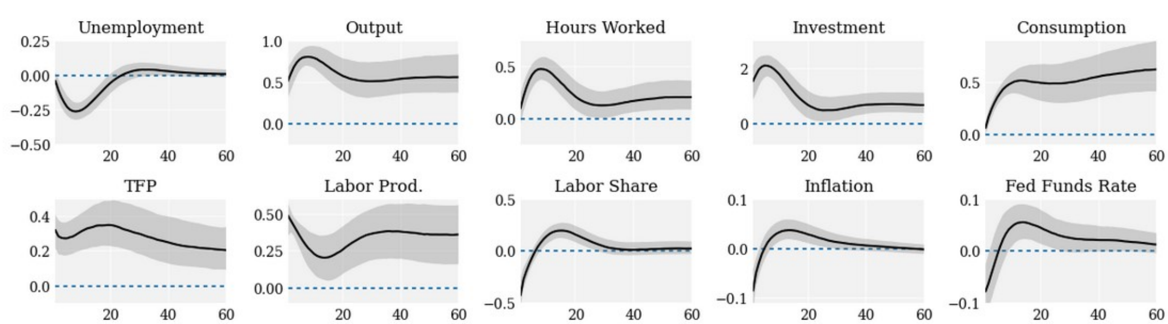
Figure 3 reports the IRFs of all the variables to the $q^{sr}_{B,t}$ shock. The shock results in co-movement of the key business cycle variables and explains significant business cycle volatility but not long-run. The impulse responses are quite significant on impact. No movement or business cycle volatility explained for TFP.

In contrast to the previous shock, this particular one exhibits no short-run or long-run movements in TFP or labor-productivity IRFs. Furthermore, the positive comovement observed in consumption, inflation, and interest rates allows us to identify this shock as a demand shock. While the previous supply shock was characterized as a long-run TFP shock, the precise interpretation of this structural shock is model-dependent. In Section IV, I will present arguments, drawing from SW07, medium-scale DSGE model, to support the notion that the short-run business cycle shock can be further labeled as a risk-premia shock. However, it is essential to acknowledge that the further structural interpretation of the demand shock varies depending on the perspective of the model utilized.

The main business cycle shock within the ACD framework is essentially a linear combination of the above two shocks. This MBC shock accounts for 26% of long-run TFP volatility, in contrast to the 53% attributed to the long-term supply shock and 17% for the business cycle fluctuations in the federal funds rate, compared to the 52.5% explained by the identified short-term demand shock. Section C in the appendix offers a bivariate example that elucidates why this MBC shock is likely to be a linear combination of the identified business cycle shocks. The example explains how this combination can result in higher volatility for the targeted variable while reducing volatility for variables moving in opposite directions due to the two orthogonal shocks.

While the two identified business cycle shocks account for approximately 99% of the business cycle volatility in GDP, they explain only about 50% of the long-run volatility in GDP, Consumption, Investment, and productivity measures. This discrepancy suggests that a significant portion of long-run fluctuations, captured by the residual in the identification ($\epsilon^{long-run}_{NB,t}$), is not associated with business cycles. I label the shocks driving these fluctuations, 'long-run non-business cycle' (LR-NBC) shocks. In the next section, I argue the presence of these non-business cycle fluctuations in the data-generating process challenges the parameter estimation of medium-scale DSGE models when using a full information setting.

Figure 2: Impulse Response Functions to Supply Shock



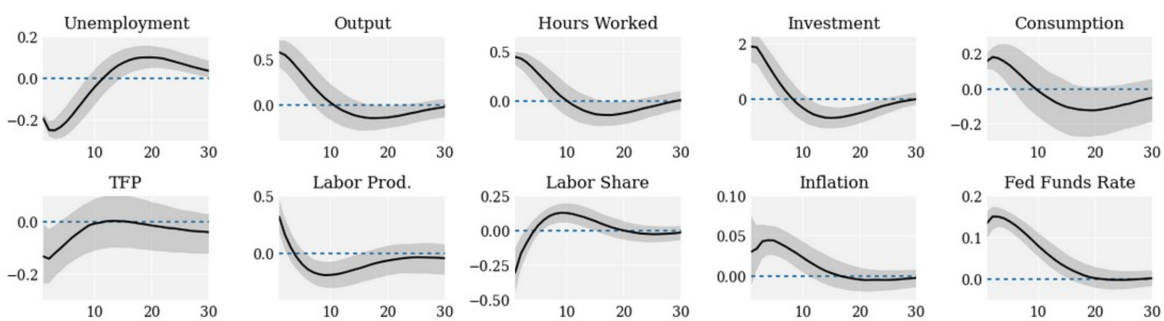
Impulse Response Functions of all the variables to the identified long-run supply shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 1: Supply Shock, Variance Contributions

	u	Y	h	I	C
Short run (6–32 quarters)	32.0	53.1	29.8	40.6	32.7
	[21.2, 43.8]	[33.4, 70.5]	[21.2, 40.2]	[25.7, 57.1]	[25.6, 39.9]
Long run (80–400 quarters)	35.7	51.8	20.6	47.9	51.4
	[19.1, 55.5]	[26.7, 72.7]	[4.8, 48.2]	[22.4, 70.2]	[26.1, 71.9]
	TFP	Y/h	wh/Y	Δp	FFR
Short run (6–32 quarters)	13.7	36.6	30.3	19.0	19.3
	[6.5, 23.7]	[26.4, 45.2]	[15.2, 42.9]	[10.7, 29.2]	[9.2, 36.0]
Long run (80–400 quarters)	53.7	54.3	41.9	14.2	15.9
	[29.4, 71.4]	[29.0, 72.1]	[18.0, 63.0]	[5.8, 29.9]	[6.4, 34.7]

Notes: Variance contributions of the identified long-run supply shock at two frequency bands. The first row (Short run) corresponds to 6–32 quarters, and the second row (Long run) to 80 quarters and beyond. 68 percent highest posterior density intervals are in brackets. Variable notation follows Section I.

Figure 3: Impulse Response Functions to Demand shock



Impulse Response Functions of all the variables to the identified short-run demand shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 2: Demand Shock, Variance Contributions

	u	Y	h	I	C
Short run (6–32 quarters)	48.6	45.8	40.9	44.2	23.0
	[36.8, 59.2]	[28.4, 65.3]	[29.9, 49.2]	[27.9, 59.5]	[16.3, 31.3]
Long run (80–400 quarters)	5.1	0.2	2.3	0.4	0.15
	[1.8, 13.4]	[0.03, 1.0]	[0.5, 8.8]	[0.08, 1.9]	[0.02, 0.8]
	TFP	Y/h	wh/Y	Δp	FFR
Short run (6–32 quarters)	7.4	22.5	16.5	10.9	50.5
	[2.5, 15.6]	[12.5, 33.9]	[7.0, 32.1]	[5.6, 18.8]	[36.6, 60.7]
Long run (80–400 quarters)	0.04	0.04	1.1	3.7	12.9
	[0, 0.2]	[0.01, 0.2]	[0.2, 4.0]	[1.1, 9.7]	[4.6, 26.3]

Notes: Variance contributions of the identified short-run demand shock at two frequency bands. The first row (Short run) corresponds to 6–32 quarters, and the second row (Long run) to 80 quarters and beyond. 68 percent highest posterior density intervals are shown in brackets. Variable definitions follow Section I.

III Full Information Estimation: Challenges

This section demonstrates how full-information estimation in the presence of long-run shocks, which do not generate business cycles, faces inherent challenges. I analytically show that the log-likelihood function of a standard linearized DSGE model reaches its maximum at the true parameters if and only if the volatility of these non-business cycle shocks is zero, establishing that their presence induces a downward bias in parameter estimation.

The argument is presented in two steps. First, I express the log-likelihood function of state-space models in the frequency domain, which decomposes a variable's fluctuations across different periodicities. This decomposition is essential, as it allows us to distinguish between long-term and short-term fluctuations. I then demonstrate that maximizing the log-likelihood in the frequency domain can be reframed as minimizing a sum of weighted ratios across all frequencies. In each ratio, the numerator consists of data-implied volatility weights, while the denominator contains model-implied volatility at the corresponding frequency. The presence of long-run non-business cycle fluctuations in the data allocates greater weight to long-run frequencies.

Second, due to cross-frequency restrictions in DSGE models, parameter adjustments that increase model-implied long-run volatility inherently decrease volatility at business cycle frequencies. Consequently, to satisfy the minimization condition, parameter estimation shifts to generate higher model-implied volatility at long-run frequencies, resulting in a downward bias in business cycle volatility estimates to account for the higher long-run volatility.

Let's start with a canonical representation of a linearized DSGE model equilibrium conditions:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Psi\epsilon_t + \Pi\eta_t \tag{14}$$

Where 1) S_t is a vector of model variables that include (i) the endogenous variables, (ii) the conditional expectations, (iii) the variables from exogenous processes if they are serially correlated; 2) ε_t is a vector of exogenous disturbances; 3) η_t is a vector of expectation errors satisfying $E_{t-1}\eta_t = 0$ for all t; 4) Γ_0, Γ_1 and Π are coefficient matrices; 5) Ψ a diagonal matrix with standard deviations of the exogenous disturbances.

Assuming the above set of equilibrium conditions that represent optimality conditions have a state-space representation and mapping to a vector of observables $\mathbf{Y}_t$:

$$S_t = \Theta_1(\theta)S_{t-1} + \Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t \tag{15}$$

$$Y_t = A(L)S_t = A(L)\left(I - \Theta_1(\theta)L\right)^{-1}\Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t \tag{16}$$

Where 1) θ_1 is a vector of standard deviations of exogenous shocks; 2) θ is a vector of all deep parameters of the DSGE model except the ones in θ_1. This distinction between the deep parameters is important because, as will be discussed subsequently, the standard deviations of exogenous shocks can be concentrated out of the likelihood function. This implies the estimation process first identifies the vector of parameters θ and; then, the vector θ_1 is determined based on the previously identified vector θ.

$$Y_t = \underbrace{A(L)\left(I - \Theta_1(\theta)L\right)^{-1}}_{\mathbf{D}(L;\theta)} \Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t \tag{17}$$

The model implied Spectral Density of variable 'k' in Y_t due to shock 'l' in ε_t at frequency 'ω' is represented as:

$$\mathcal{SD}(\omega, k, l; \theta, \theta_1) = \frac{1}{2\pi}\mathcal{M}(\omega, y_k, l; \theta)\sigma_l^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, l; \theta) = \left|\mathbf{D}^k(e^{i\omega}; \theta)\Theta_\epsilon^l(\theta)\right|^2 \tag{18}$$

For the sake of tractability, let's make a simplifying assumption. Let's assume the true data generating process of the variable y_k involves two exogenous shocks '**B**' & '**lNB**'. Here, '**B**' represents a scalar business cycle shock, while '**lNB**' represents a long-run shock that doesn't cause business cycles.

Since standard DSGE models don't allow both categories of long-run shocks by having restrictions that allow one to cause business cycles but not the other, the exogenous shock process vector ε in the canonical representation above comprises only a business cycle shock[2], i.e. $l = \mathbf{B}$ with model implied variance of $\tilde{\sigma}_{\mathbf{B}}^2$.

The log-likelihood function of the above state space model in the frequency domain following Harvey (1989) is as follows:

$$\log L(\theta, \theta_1) = -\sum_{j=1}^{T}\left(\log \frac{1}{2\pi}\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)\tilde{\sigma}_{\mathbf{B}}^2 + \frac{I(\omega_j, y_k)}{\frac{1}{2\pi}\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)\tilde{\sigma}_{\mathbf{B}}^2}\right) \tag{19}$$

where, $\omega_j = \frac{2\pi t}{T}$. The likelihood function depends on two key elements: the spectral density of the model, denoted as $\mathcal{SD}(\omega_j, k, l; \theta, \theta_1)$, and the periodogram $I(\omega_j, k)$, which represents the data-implied volatility at frequency ω_j. In this framework, the data-implied variances are specified for the business cycle shock and a long-run non-business cycle shock as $\sigma_{\mathbf{B}}^2$ and $\sigma_{\mathbf{lNB}}^2$, respectively.

$$I(\omega_j, k) = \frac{1}{2\pi}\mathcal{D}(\omega_j, y_k, \mathbf{B})\sigma_{\mathbf{B}}^2 + \frac{1}{2\pi}\mathcal{D}(\omega_j, y_k, \mathbf{lNB})\sigma_{\mathbf{lNB}}^2 \tag{20}$$

where as defined in section A, $\mathcal{D}(\omega_j, y_k, l) = q_l'\overline{D^k\left(e^{-i\omega_j}\right)}D^k\left(e^{-i\omega_j}\right)q_l$

Maximising log L with respect to $\sigma_{\mathbf{B}}^2$ gives:

$$\tilde{\sigma}_{\mathbf{B}}^2(\theta) = \frac{2\pi}{T}\sum_{j=1}^{T}\frac{I(\omega_j, y_k)}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)} = \frac{2\pi}{T}S(\theta) \tag{21}$$

Following, Harvey (1989) (pg. 193), the exogenous shock variance may therefore, be concentrated out of

[2]Subsection D shows how multiple shocks in the model can be mapped to a business cycle shock.

the likelihood function, with the result that maximizing $\log L$ in (19) is equivalent to minimizing $S(\theta)$, where

$$S(\theta) = \underbrace{\sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{short-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{long-run volatility}} \tag{22}$$

Substituting (20) in (22),

$$S(\theta) = \underbrace{\sum_{j=1}^{t} \frac{1}{2\pi} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B})\sigma_{\mathbf{B}}^2 + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\sigma_{\mathbf{lNB}}^2}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{short-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{1}{2\pi} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B})\sigma_{\mathbf{B}}^2 + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\sigma_{\mathbf{lNB}}^2}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{long-run volatility}} \tag{23}$$

DSGE models estimated in the time domain are equivalent to fitting the model over the whole spectral density. These models generate cross-frequency restrictions, the presence of information in the estimation that the model is not intended to explain may affect the estimates. The area under the power spectrum over the range $[-\pi, \pi]$ is equal to the variance of the process (Harvey (1989), pg. 58). More generally,

$$\underbrace{\sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}_{\text{short-run}} + \underbrace{\sum_{j=t+1}^{T} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}_{\text{long-run}} = 1 \tag{24}$$

Thus, the power spectrum may be viewed as a decomposition of the variance of the process in terms of frequency. In summary, the estimation of vector θ of model parameters is equivalent to minimizing $S(\theta)$ in (23) subject to the constraint (24).

To understand the bias introduced by the presence of long-run Non-business cycle fluctuations $\left(\sigma_{\mathbf{lNB}}^2 > 0\right)$ in the data $(I_y(\omega_j, k))$ using Bayesian likelihood estimation, we make the following three assumptions,

Assumption 1. *: Suppose* $\exists\, \theta^*$ *s.t.* $\mathcal{D}(\omega_j, y_k, \boldsymbol{B}) = \mathcal{M}(\omega_j, y_k, \boldsymbol{B}; \theta^*)\ \forall\, \omega_j$ *where* $j \in \{1, 2, \cdots, T\}$

Assumption 1 implies that, the above DSGE model is well-specified for business cycle fluctuations. This implies that there exists a vector of parameters (θ^*) such that model implied volatility due to business cycle shock $(\mathcal{SD}(\omega_j, y_k, \mathbf{B}; \theta^*))$ is equal to data implied volatility of the business cycle shock $\left(\frac{1}{2\pi}\mathcal{D}(\omega_j, y_k, \mathbf{B})\sigma_{\mathbf{B}}^2\right)$ at all frequencies ω_j.

Assumption 2. *: Suppose* $\exists\, \theta'$ *s.t.* $\frac{\mathcal{D}(\omega_j, y_k, \boldsymbol{B}) + \mathcal{D}(\omega_j, y_k, \boldsymbol{lNB})\kappa}{\mathcal{D}(y_k, \boldsymbol{B}, \boldsymbol{lNB})} = \mathcal{M}(\omega_j, y_k, \boldsymbol{B}; \theta')\ \forall\, \omega_j$ *where* $j \in \{1, 2, \cdots, T\}$

Assumption 2 implies that there exists a vector of parameters (θ') such that model implied volatility due to a business cycle shock $\left(\mathcal{SD}(\omega_j, y_k, \mathbf{B}; \theta')\right)$ is equal to data implied normalized volatility of both a business cycle and non-business cycle shock $\left(\sum_{j=1}^{t} \frac{1}{2\pi} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B})\sigma_{\mathbf{B}}^2 + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\sigma_{\mathbf{lNB}}^2}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB})}\right)$ at all frequencies ω_j.

where, $\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB}) = \prod_{j=1}^{T} \left(\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\kappa\right)$ & $\kappa = \frac{\sigma_{\mathbf{lNB}}^2}{\sigma_{\mathbf{B}}^2}$

$\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB})$ in assumption 2 is used for normalization of spectral density due to both business cycles and non-business cycle shocks such that the combined spectral density of the two shocks satisfies the Kolmogorov result[3].

Assumption 3. *:* $\frac{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, \boldsymbol{lNB})}{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, \boldsymbol{B})} > \frac{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, \boldsymbol{lNB})}{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, \boldsymbol{B})}$

While presented as an assumption, this assumption is grounded in the empirical findings detailed in Section II. In this context, the left-hand side (LHS) represents the ratio of the long-run spectral density of long-run non-business cycle shocks to business-cycle shocks, while the right-hand side (RHS) represents the ratio of the short-run spectral density of long-run non-business cycle shocks to business-cycle shocks. Given that the short-run spectral density induced by long-run non-business cycle shocks $\left(\sum_{j=1}^{t} \mathcal{D}\left(\omega_j, y_k, \mathbf{lNB}\right)\right) \approx 0$ and the long-run spectral density $\left(\sum_{j=t+1}^{T} \mathcal{D}\left(\omega_j, y_k, \mathbf{lNB}\right)\right) \approx 50\%$, while the short-run spectral density due to business cycle shocks $(\sum_{j=1}^{t} \mathcal{D}\left(\omega_j, y_k, \mathbf{B}\right))$ is greater than the long-run spectral density due to business cycle shocks $(\sum_{j=t+1}^{T} \mathcal{D}\left(\omega_j, y_k, \mathbf{B}\right))$, it follows that LHS > RHS.

Theorem 1. *: Under assumptions 1, 2 & 3, the minimization of $S(\theta)$ is achieved at true parameters θ^* if and only if $\sigma^2_{\boldsymbol{lNB}} = 0$.*

Appendix Section A provides the detailed proof for the theorem and specifies the minimum values of $S(\theta)$. Here, I will highlight additional implications arising from this theorem. By filtering out non-business cycle fluctuations, the minimum value of $S(\theta)$ is attained at the true parameter values θ^*, where $S(\theta^*) = \frac{T\sigma^2_{\mathbf{B}}}{2\pi}$. Based on the result from Equation (21), the estimated volatility of the business-cycle shock ($\tilde{\sigma}^2_{\mathbf{B}}$) equals the true volatility when there is no long-run non-business cycle volatility $\sigma^2_{\mathbf{lNB}} = 0$.

$$\tilde{\sigma}^2_{\mathbf{B}}(\theta^*) = \frac{2\pi}{T} S(\theta^*) = \sigma^2_{\mathbf{B}}$$

However, when substantial long-run non-business cycle volatility is present $\left(\sigma^2_{\mathbf{lNB}} > 0\right)$, the minimum value of $S(\theta)$ is attained at alternative parameters θ', where $S(\theta') = \frac{T\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB})}{2\pi}$. Consequently, the estimated volatility of the business-cycle shock is upwardly biased since $\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB}) > 1$.

$$\tilde{\sigma}^2_{\mathbf{B}}(\theta') = \frac{2\pi}{T} S(\theta') = \sigma^2_{\mathbf{B}} \mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB})$$

The bias in the parameters would also result in a downward bias in model-implied business cycle implications when estimated in a full information setting with significant long-run non-business cycle fluctuations. Given the inequality (27) & cross-frequency restrictions in (24),

$$\sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta') < \sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta^*) = \sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, \mathbf{B})$$

This implies that the presence of long-run non-business cycle shocks would result in parameters such that the model-implied short-run volatility $\left(\sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta')\right)$ would be lower than the actual data implied short-run volatility $\left(\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, \mathbf{B})\right)$.

[3] $\sum_{j=1}^{T} \log \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\kappa}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB})} = 0$

$$\sigma^2_{\mathbf{INB}}\uparrow \xrightarrow{minimize \;\; S(\theta)} \underbrace{\sum_{j=t+1}^{T} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)\uparrow}_{\text{long-run}} \xrightarrow{restriction} \underbrace{\sum_{j=1}^{t} \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)\downarrow}_{\text{short-run}}$$

For intuition, as the volatility ($\sigma^2_{\mathbf{INB}}$) of long-run non-business cycle shocks increases, the minimization of the objective function $S(\theta)$ results in a θ such that the model implied long-run volatility increases given the cross-frequency restrictions resulting in a downward bias on the short-run volatility of the model. Given the evidence of a significant fraction of long-run non-business cycle fluctuations from the previous section, this argues for estimation in a limited information setting via IRF matching with the identified business cycle shocks in section II. The following section showcases the normative and policy implications of the SW07 by comparative analysis of the model, estimated using conditional moments, and the Bayesian likelihood estimation of the original estimated model.

IV Application: Smets & Wouter (2007)

This section centers on applying the above identification strategy for comparative analysis with full information estimation using a benchmark medium-scale DSGE model proposed by SW07. The model serves a two-fold purpose: firstly, to label the demand shock based on both the model and empirical IRFs; secondly, to examine the normative and policy implications by identifying the model parameters through impulse response matching, a method commonly utilized, among others, in Christiano et al. (2005).

The SW07 model features real and nominal rigidities including investment adjustment costs, habit persistence, variable capacity utilization, and partial indexation of sticky prices and wages under monopolistic competition. It evolves along a balanced growth path with variables expressed as deviations from steady state. Business cycle dynamics are driven by seven shocks: five AR(1) processes (TFP, investment-specific technology, government spending, risk premium, and monetary policy) and two ARMA(1,1) markup shocks to wages and prices, introducing rich autocorrelated disturbances. The log-linearized equilibrium conditions for the model are presented in the appendix F.

A Identification Analysis

The model is estimated using data on seven variables: real GDP, real consumption, real investment, real wage, inflation, hours worked and the nominal interest rate.

For comparative analysis with the dissection strategy suggested in this paper, the SW07 model is estimated with Bayesian likelihood estimation techniques in SW07 using seven key macroeconomic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment, and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Then, the business cycle shock identification strategy is applied to a VAR with the same seven observables.

The impulse response results, reflecting both long-run and short-run shocks, consistently align with the identification presented in the benchmark VAR model introduced by ACD in the preceding section. The identification process is substantiated by the negative conditional correlation observed between real

GDP and inflation, the long-run persistence of the real GDP impulse response as depicted in Figure 4, and the substantial explanatory power exhibited by the macroeconomic variables' business cycle and long-run volatility, as demonstrated in Table 3. This alignment highlights the identification of the long-run shock being attributed to a supply shock.

Similarly, the identification of the short-run shock as a demand shock is reinforced by the positive conditional correlation observed between real GDP, inflation, and the federal funds rate. These conditional correlations are evident in Figure 4. Moreover, the explanation of the identified model to explain the business-cycle volatility across various macro-variables, highlighted in Table 3, further substantiates the identification of the short-run shock as a demand shock.

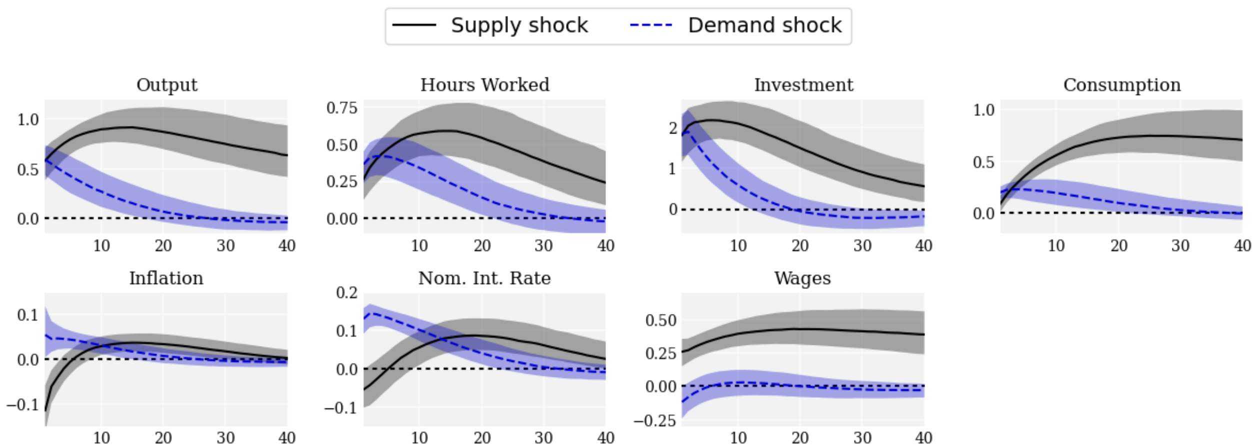
Given the array of seven shocks in SW07, there are only two shocks with potential to generate characterstic business-cycle comovement. Within the category of demand shocks that comprises, discount factor shock, risk-premia shock, monetary shock, and investment-specific technology shock, it is the risk premium shock that helps to explain the comovement of consumption and investment in presence of nominal rigidities. This encourages the further identification of the demand shock as a risk premia shock. While within the category of supply shocks, that includes total factor productivity (TFP) shocks, wage and price mark-up shocks, it is the TFP shock that would be able to replicate the desired long-run fluctuations and the comovement.

In the following subsection, I focus on parameter identification by employing impulse response matching with the same set of observables used in SW07. This approach classifies the previously identified long-run shock as a TFP shock and the short-run demand shock as a risk-premia shock. This methodology rests on the implicit assumption that the identified business cycle shocks are structural, rather than a reduced form linear combination of underlying structural shocks. While this assumption is strong—particularly for the identification strategy suggested in the paper, which makes no assumption about the structural origins but relies on the persistence of shocks—it serves as a practical trade-off to address certain controversial assumptions embedded within DSGE models, as highlighted by Chari et al. (2009) (CKM09).

CKM09 critique the New Keynesian tradition, arguing that it often leads to models that cannot be reliably used for policy analysis. They contend that the drive to fit macroeconomic data has resulted in researchers adding numerous shocks and other features to improve macro-level fit, using the same aggregate data to estimate the corresponding parameters. However, this approach often lacks the discipline provided by microeconomic evidence, leading to the proliferation of free parameters in New Keynesian models. CKM09 specifically critique the SW07 model, questioning the structural nature of four of its shocks: wage markups, price markups, exogenous spending, and risk premia. They argue that introducing wage and price markup shocks amounts to mechanically inserting a labor wedge into the model, which can be interpreted in various ways.

By assuming the identified business cycle shocks are structural and utilizing their conditional correlations, I eliminate five of the seven shocks in SW07, thereby reducing the parameter space by ten. This helps avoid three of the questionable shocks identified by CKM09, although the risk-premia shock remains, suggesting the need for further structural refinement in DSGE models for future improvements.

Figure 4: Smets & Wouter (2007) VAR: Impulse Response Functions



Impulse Response Functions of all the variables to the identified supply shock. Horizontal axis: time horizon in quarters. Shaded area: 68 percent Highest Posterior Density Interval (HPDI).

Table 3: Variance Contributions

	Y	h	I	C
Supply Shock: Short run (6–32 quarters)	57.3	23.8	42.9	26.7
	[33.9, 76]	[10.5, 36.1]	[22.7, 60.3]	[18.7, 36.6]
Supply Shock: Long run (80–400 quarters)	66.5	69.8	69.1	65.0
	[36.9, 86.4]	[44.8, 84.4]	[40, 86.5]	[34.9, 85.3]
Demand Shock: Short run (6–32 quarters)	42.5	30.7	38.4	17.9
	[23.6, 65.7]	[16, 45.6]	[20.4, 59.8]	[8.3, 27.8]

	FFR	w	Δp
Supply Shock: Short run (6–32 quarters)	15.3	19.8	22.2
	[6.3, 30.9]	[10.5, 32.5]	[10.2, 37.9]
Supply Shock: Long run (80–400 quarters)	27.7	65.3	27.7
	[12.9, 51.7]	[33.8, 85.2]	[11, 56.0]
Demand Shock: Short run (6–32 quarters)	41.2	5.5	8.3
	[25.4, 53.6]	[2.3, 12.7]	[3.1, 20.9]

Notes: Variance contributions of the identified business cycle shocks at two frequency bands. The first row (Short run) corresponds to 6–32 quarters, and the second row (Long run) to 80 quarters and beyond. 68 percent HPDIs are shown in brackets. Variables are defined in Section I.

B Comparative Analysis of Estimation Methods

Figure 5 presents impulse response functions (IRFs) to illustrate the effects of different estimation methods on a model's performance. The blue lines depict the IRFs of the model estimated through IRF matching with identified business cycle shocks (shown in red) in response to a one-standard-deviation Total Factor

Productivity (TFP) shock and a risk-premia shock. This estimation method accurately captures the dynamic responses of these identified shocks. Additionally, I include the IRFs of the SW07 model estimated using Bayesian likelihood, represented in black. While the Bayesian estimation aligns well with the empirical IRFs in most aspects, there are disparities in investment and hours worked.

To comprehend the implications of these two estimation methods, this section illustrates the comparative analysis in two parts.

Firstly, figure 6 focuses on the response to a long-run TFP shock. Here, the two estimation methods lead to significantly different normative and policy implications. In the Bayesian estimated model (in black), inflation and the output gap exhibit a positive correlation, resembling the divine coincidence often seen in New Keynesian models with demand shocks. In contrast, the model estimated via IRF matching with identified business cycle shocks reveals a negative conditional correlation between the output gap and inflation. This suggests a policy tradeoff for the monetary authority: lowering interest rates to counter falling inflation may lead to a further increase in the output gap.

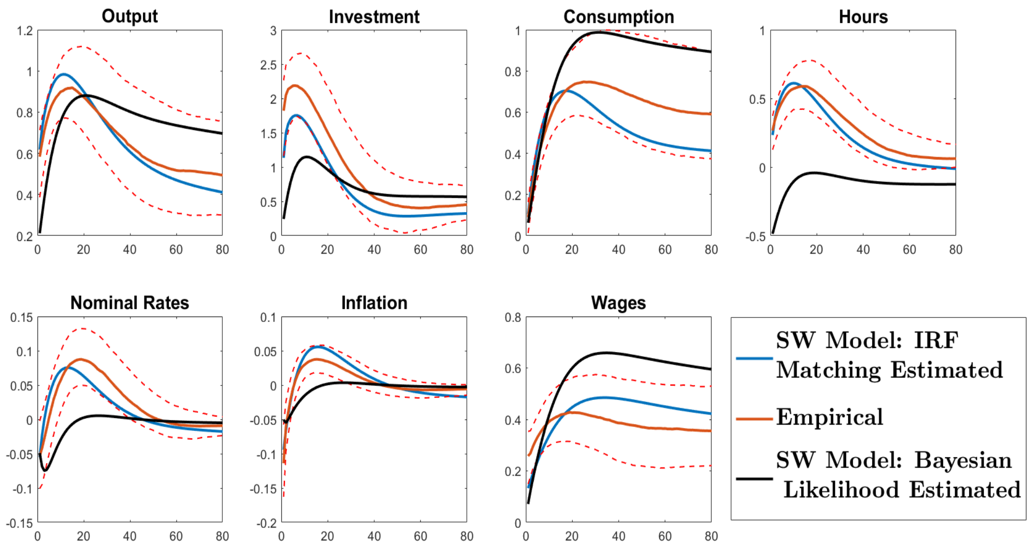
The differences in the conditional correlation between the output gap and inflation can be attributed to variations in key parameter values, specifically wage indexation, the Calvo wage parameter, price indexation, and the Calvo price parameter. SW07, following Christiano et al. (2005), incorporate backward price indexation into their models, where firms unable to adjust prices in a given period mechanically align them with past inflation. This assumption leads to differing dynamics in price and wage rigidity between estimation methods.

As shown in Table 4, IRF matching, compared to Bayesian likelihood estimation, results in higher wage indexation and a larger Calvo wage parameter, leading to greater wage rigidities relative to price rigidities. This disparity explains the differing conditional correlations between the output gap and inflation across the two estimation methods, particularly observable in the response of hours worked and investment to a total factor productivity (TFP) shock (Figure 5a). Under Bayesian likelihood estimation, the greater price rigidities lead to a smaller decline in prices following an expansionary TFP shock, causing a limited increase in demand as TFP substitutes for hours worked, resulting in a fall in hours worked and a negative output gap. In contrast, IRF matching's higher wage rigidities incentivize firms to hire more labor hours, as wage stickiness makes it optimal to increase labor hours and output above the flexible level. This allows firms to increase current production and save for future investment, anticipating higher wages due to the wealth effects of a long-run TFP shock.

This also addresses the another criticism raised by CKM09 regarding the assumption of backward price indexation, which they argue is counterfactual. Based on studies such as Bils and Klenow (2004), Golosov and Robert E. Lucas (2007), Midrigan (2010), and Nakamura and Steinsson (2008), that provide micro-level evidence on price behavior, they argue that backward price indexation is inconsistent with the data. In models using full information, the inclusion of backward price indexation forces the model to fit persistent long-run inflation, mechanically generating inflation persistence. However, this mechanism is inconsistent with the empirical micro data. CKM09 emphasize that this backward indexation feature influences policy advice by exaggerating the costs of disinflation. If inflation persistence were driven by a different mechanism, the costs of disinflation could be significantly lower. As illustrated in Table 4, the price indexation parameter under the IRF matching approach is close to zero, aligning with microeconomic evidence. This contrasts with models estimated under full information, where price indexation persists and contributes to long-run inflation. By focusing on business cycle shocks rather than non-business cycle fluctuations, the IRF matching method yields more accurate estimates, reducing reliance on counterfactual assumptions like backward price indexation.

Figure 5: Smets & Wouters (2007): Impulse Response Functions

(a) Long-run TFP Shock



(b) Risk Premia Shock

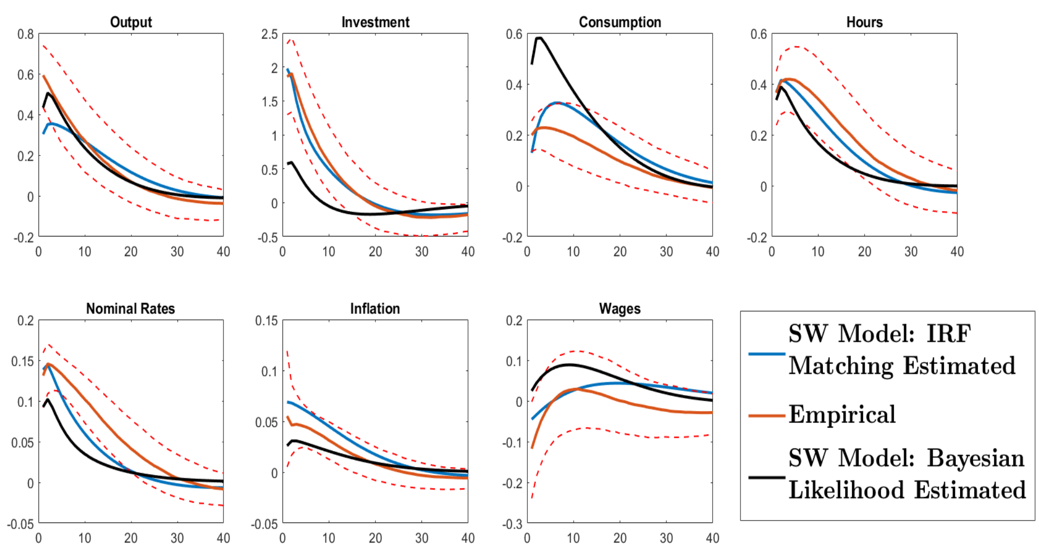


Table 4: Estimated Parameter Values

Parameter	Description	IRF Matching	Bayesian Likelihood
ρ_{ga}	Feedback technology on exogenous spending	0.9905	0.2272
$100(\beta^{-1}-1)$	Time preference rate (percent)	1.7162	0.1239
α	Capital share	0.1780	0.2079
ψ	Capacity utilization cost	0.9658	0.6723
Θ	Investment adjustment cost	0.0145	8.0415
σ_c	Risk aversion	1.5866	1.3295
λ	External habit degree	0.6084	0.8789
Θ	Fixed cost share	1.0000	1.4888
l_w	Indexation to past wages	0.8241	0.5542
ξ_w	Calvo parameter wages	0.8600	0.8682
l_p	Indexation to past prices	0.0000	0.2127
ξ_p	Calvo parameter prices	0.7200	0.7697
σ_l	Frisch elasticity	0.2500	2.2934
r_π	Taylor rule inflation feedback	1.0000	1.7822
$r_{\triangle y}$	Taylor rule output growth feedback	0.3835	0.0010
r_y	Taylor rule output level feedback	0.0640	0.1907
ρ	Interest rate persistence	0.7522	0.8283
ρ_a	Persistence productivity shock	0.9974	0.9975
ρ_b	Persistence risk premium shock	0.8300	0.2751
ρ_g	Persistence spending shock	0.9795	0.9810
γ	Growth rate	1.0000	1.0032
σ_a	Std. productivity shock	0.4247	0.5557
σ_b	Std. risk premium shock	0.0131	0.1762

Figure 6: Long-run TFP Shock: Output Gap & Inflation

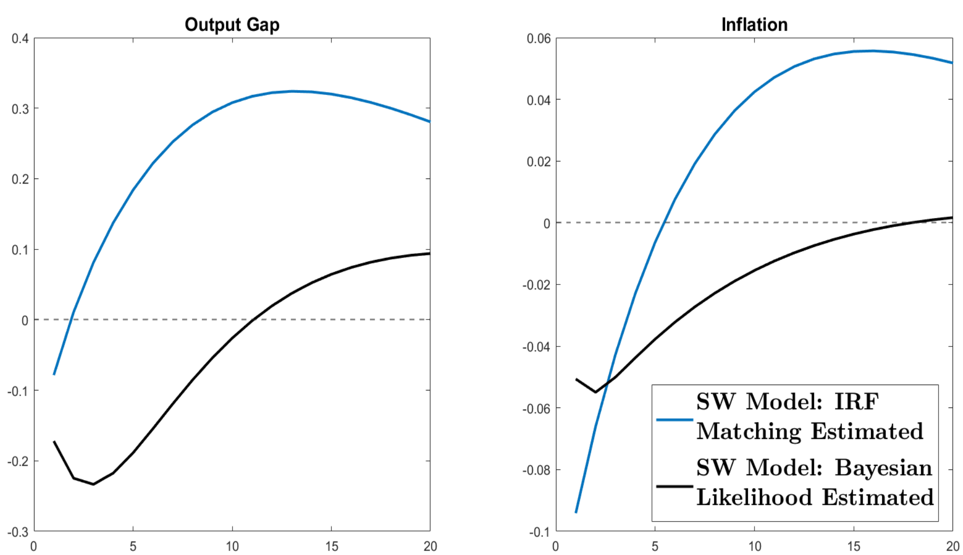
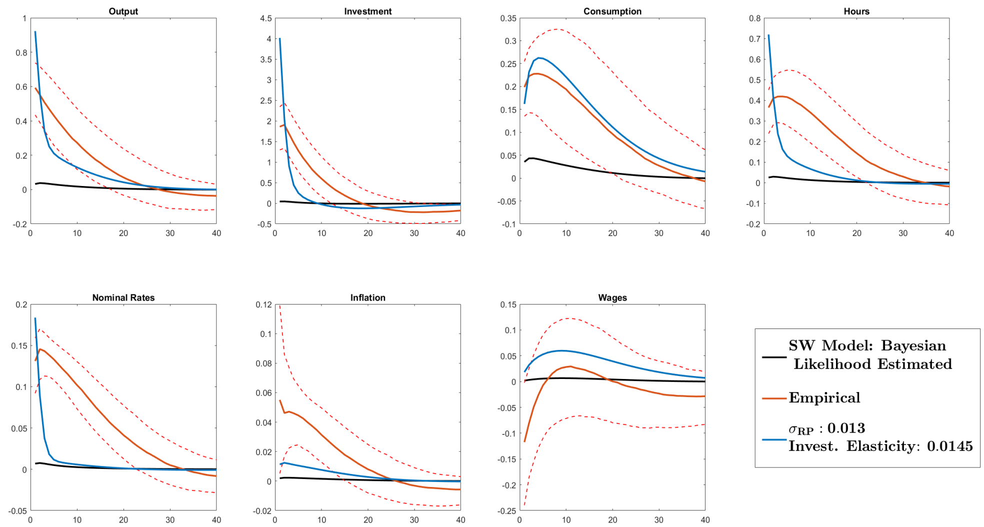


Figure 7: Risk Premia Shock: Internal vs. External Propagation



Secondly, through Figure 7, I argue the two estimations result in different inferences about the underlying internal and external propagation mechanisms of SW07. The mentioned figure has empirical and model IRFs for a risk premia shock. The standard deviation of the risk premia shock (σ_{rp}) for the Bayesian likelihood estimated is 0.1762. I first replaced it with the IRF matching estimated σ_{rp} of 0.0131 which is 17 times smaller than the full information setting value while I kept the rest of the parameters estimated by Bayesian likelihood estimation unchanged. As one may observe in the black IRFs, in the same figure, the risk premia shock becomes insignificant. Next I replace the investment elasticity estimated by Bayesian likelihood (8.0145) with the IRF matching estimated value of 0.0145 and as we may observe risk premia IRFs become significant on impact and persistence. This showcases the key differences in estimation where full information setting argues for a stronger external propagation role in risk premia-driven business cycles while IRF matching argues for a stronger internal propagation through financial frictions.

In summary, the two estimations result in key differences for the same model over the same dataset in terms of normative and policy implications which is crucial given the use of augmented versions of such medium-scale DSGE models at central banks. Based on theoretical insights from the previous section, it argues for limited information estimation to avoid any biased business cycle implications because of non-business cycle information in the full information settings.

V Conclusion

This paper introduces a novel approach to identifying the primary sources of business cycle fluctuations without relying on restrictive ex-ante assumptions about the underlying drivers of these cycles. In contrast to a single-shock assumption, this framework allows for multiple business cycle shocks, enabling a more flexible identification process. By testing and relaxing the single-shock assumption, the analysis demonstrates that the classic dichotomy between supply and demand shocks offers a more accurate explanation of macroeconomic covariances at the business cycle frequency.

The empirical findings show that the two identified business cycle shocks account for nearly all business cycle volatility in GDP, as well as significant volatility in key macroeconomic variables such as consumption, investment, and unemployment. Additionally, the results reveal the presence of a second class of long-run supply shocks that do not contribute to GDP's business cycle fluctuations. This distinction underscores a key theoretical insight: parameter bias is inherent in medium-scale DSGE models estimated under full-information settings, leading to downward distortions in the model-implied business cycle volatility.

By estimating medium-scale DSGE models using conditional moments derived from the identified business cycle shocks, the paper compares the normative and policy implications of the same model under both full-information and limited-information settings. Estimating DSGE models in a limited-information setting, using moments relevant to the business cycle, addresses several critiques of New Keynesian models and enhances the validity of the proposed identification strategy. Ultimately, this paper offers a robust framework for disentangling business cycle fluctuations from non-business cycle fluctuations, thereby improving the reliability of model estimation and policy analysis in macroeconomic models.

A Theorem 1

Under assumptions 1, 2 & 3, the minimization of $S(\theta)$ is achieved at true parameters θ^* if and only if $\sigma^2_{\mathbf{INB}} = 0$.

Proof. First, suppose $\sigma^2_{\mathbf{INB}} = 0$,

From assumption 1, θ^* is s.t. $\mathcal{D}(\omega_j, y_k, \mathbf{B}) = \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta^*)\ \forall\ \omega_j$ where $j \in \{1, 2, \cdots, T\}$

Substituting $\mathcal{D}(\omega_j, y_k, \mathbf{B})$ with $\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta^*)$ for all frequencies in the minimization problem (23)

$$S(\theta^*) = T\frac{\sigma^2_{\mathbf{B}}}{2\pi} + \sum_{j=1}^{T} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{INB})\,\sigma^2_{\mathbf{INB}}}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta^*)}$$

Given $\sigma^2_{\mathbf{INB}} = 0, \therefore S(\theta^*) = T\frac{\sigma^2_{\mathbf{B}}}{2\pi}$

Following Lemma 1, $S(\theta)$ is minimized at θ^* for $\sigma^2_{\mathbf{INB}} = 0$.

Next, suppose $\sigma^2_{\mathbf{INB}} > 0$,

Let $\sigma^2_{\mathbf{INB}} = \kappa\ \sigma^2_{\mathbf{B}}$, substituting $\sigma^2_{\mathbf{INB}}$ in (23)

$$S(\theta) = \frac{\sigma^2_{\mathbf{B}}}{2\pi}\left(\underbrace{\sum_{j=t+1}^{T} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{long-run volatility}} + \underbrace{\sum_{j=1}^{t} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{short-run volatility}}\right) \tag{25}$$

Divide and multiply (25) with $\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})$

where, $\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB}) = \prod_{j=1}^{T} \left(\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa\right)$

$$S(\theta) = \frac{\sigma^2_{\mathbf{B}}\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})}{2\pi}\left(\underbrace{\sum_{j=t+1}^{T} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{long-run volatility}} + \underbrace{\sum_{j=1}^{t} \frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta)}}_{\text{short-run volatility}}\right) \tag{26}$$

From assumption 2, θ' is s.t. $\frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})} = \mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta')\ \forall\ \omega_j$ where $j \in \{1, 2, \cdots, T\}$

Substituting $\frac{\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{INB})\kappa}{\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})}$ with $\mathcal{M}(\omega_j, y_k, \mathbf{B}; \theta')$ for all frequencies in the minimization problem (26)

$$S(\theta') = T\frac{\sigma^2_{\mathbf{B}}\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})}{2\pi}$$

From AM-GM inequality, the minimum value of $S(\theta)$ in (26) is $T\frac{\sigma^2_{\mathbf{B}}\mathcal{D}(y_k, \mathbf{B}, \mathbf{INB})}{2\pi}$

Thus, $S(\theta)$ in the presence of both business and non-business cycle shocks is minimized at θ'.

Assumption 3 implies the following inequality,

$$\frac{\sum_{j=t+1}^{T}\mathcal{D}(\omega_j,y_k,\mathbf{B})+\mathcal{D}(\omega_j,y_k,\mathbf{INB})\kappa}{\sum_{j=1}^{t}\mathcal{D}(\omega_j,y_k,\mathbf{B})+\mathcal{D}(\omega_j,y_k,\mathbf{INB})\kappa} > \frac{\sum_{j=t+1}^{T}\mathcal{D}(\omega_j,y_k,\mathbf{B})}{\sum_{j=1}^{t}\mathcal{D}(\omega_j,y_k,\mathbf{B})}$$

Dividing numerator with $\sum_{j=1}^{t}\mathcal{D}(\omega_j,y_k,\mathbf{B})$ and denominator with $\sum_{j=t+1}^{T}\mathcal{D}(\omega_j,y_k,\mathbf{B})$ on both sides

$$\frac{1+\frac{\sum_{j=t+1}^{T}\mathcal{D}(\omega_j,y_k,\mathbf{INB})\kappa}{\sum_{j=t+1}^{T}\mathcal{D}(\omega_j,y_k,\mathbf{B})}}{1+\frac{\sum_{j=1}^{t}\mathcal{D}(\omega_j,y_k,\mathbf{INB})\kappa}{\sum_{j=1}^{t}\mathcal{D}(\omega_j,y_k,\mathbf{B})}} > 1$$

Following assumptios 1 & 2, substituting $\frac{\mathcal{D}(\omega_j,y_k,\mathbf{B})+\mathcal{D}(\omega_j,y_k,\mathbf{INB})\kappa}{\mathcal{D}(y_k,\mathbf{B},\mathbf{INB})}$ with $\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta^{'})$ & $\mathcal{D}(\omega_j,y_k,\mathbf{B})$ with $\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta^{*})$ for all frequencies in the above inequality

$$\frac{\sum_{j=t+1}^{T}\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta')}{\sum_{j=1}^{t}\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta')} > \frac{\sum_{j=t+1}^{T}\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta^{*})}{\sum_{j=1}^{t}\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta^{*})} \tag{27}$$

Therefore, $\theta' \neq \theta^{*}$ □

B Minimum Value of the Objective Function

Lemma 1. *: The minimum value of $S(\theta)$ when $\sigma^2_{\mathbf{INB}}=0$ is $T\frac{\sigma^2_{\mathbf{B}}}{2\pi}$.*

$$S(\theta)=\underbrace{\sum_{j=1}^{t}\frac{1}{2\pi}\frac{\mathcal{D}(\omega_j,y_k,\mathbf{B})\sigma^2_{\mathbf{B}}+\mathcal{D}(\omega_j,y_k,\mathbf{INB})\sigma^2_{\mathbf{INB}}}{\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta)}}_{\text{long-run volatility}}+\underbrace{\sum_{j=t+1}^{T}\frac{1}{2\pi}\frac{\mathcal{D}(\omega_j,y_k,\mathbf{B})\sigma^2_{\mathbf{B}}+\mathcal{D}(\omega_j,y_k,\mathbf{INB})\sigma^2_{\mathbf{INB}}}{\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta)}}_{\text{short-run volatility}} \tag{28}$$

Let

$$x_j=\frac{\mathcal{D}(\omega_j,y_k,\mathbf{B})}{\mathcal{M}(\omega_j,y_k,\mathbf{B};\theta)} \tag{29}$$

when $\sigma^2_{\mathbf{INB}}=0$

$$S(\theta)=\frac{\sigma^2_{\mathbf{B}}}{2\pi}\sum_{j=1}^{T}x_j \tag{30}$$

AM-GM Inequality: For any list of T nonnegative real numbers $x_1,x_2,\ldots,x_T$,

$$\frac{x_1+x_2+\cdots+x_T}{T} \geq \sqrt[n]{x_1\cdot x_2\cdots x_T} \tag{31}$$

and that equality holds if and only if $x_1=x_2=\cdots=x_T$.

Therefore,

$$S(\theta) \geq \frac{T\sigma_{\mathbf{B}}^2}{2\pi} \sqrt[n]{x_1 \cdot x_2 \cdots x_T} \tag{32}$$

$$S(\theta) = \frac{T\sigma_{\mathbf{B}}^2}{2\pi} \tag{33}$$

iff $x_1 = x_2 = \cdots = x_T = 1$

C MBC of ACD

Consider our simple bivariate VAR

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{\text{supply}} \\ \varepsilon_t^{\text{demand}} \end{bmatrix} \tag{34}$$

The Impulse response can be computed recursively as follows

$$\begin{cases} IR_t = Bs & \text{for } t = 0 \\ IR_t = \Phi IR_{t-1} & \text{for } t = 1, \ldots, h \end{cases} \tag{35}$$

$$\begin{bmatrix} IR_0^y \\ IR_0^r \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \tag{36}$$

$$\begin{cases} \text{VD}_{y_0}^{\varepsilon^{\text{supply}}} & = \frac{b_{11}^2}{b_{11}^2+b_{12}^2} \\ \text{VD}_{y_0}^{\varepsilon^{\text{demand}}} & = \frac{b_{12}^2}{b_{11}^2+b_{12}^2} \end{cases} \qquad \begin{cases} \text{VD}_{r_0}^{\varepsilon^{\text{supply}}} & = \frac{b_{21}^2}{b_{21}^2+b_{22}^2} \\ \text{VD}_{r_0}^{\varepsilon^{\text{demand}}} & = \frac{b_{22}^2}{b_{21}^2+b_{22}^2} \end{cases} \tag{37}$$

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} q_{11} & q_{21} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{\text{supply}} \\ \varepsilon_t^{\text{demand}} \end{bmatrix} \tag{38}$$

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} q_{11}\varepsilon_t^{\text{supply}} + q_{21}\varepsilon_t^{\text{demand}} \\ q_{12}\varepsilon_t^{\text{supply}} + q_{22}\varepsilon_t^{\text{demand}} \end{bmatrix} \tag{39}$$

$$\begin{bmatrix} y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{\text{MBC}} \\ \varepsilon_t^{\text{SBC}} \end{bmatrix} \tag{40}$$

$$\begin{cases} \text{VD}_{y_0}^{\varepsilon^{\text{MBC}}} & = \frac{q_{11}^2 b_{11}^2 + q_{21}^2 b_{12}^2 + 2q_{11}q_{21}b_{11}b_{12}}{b_{11}^2+b_{12}^2} \\ \text{VD}_{y_0}^{\varepsilon^{\text{SBC}}} & = \frac{q_{12}^2 b_{11}^2 + q_{22}^2 b_{12}^2 + 2q_{12}q_{22}b_{11}b_{12}}{b_{11}^2+b_{12}^2} \end{cases} \qquad \begin{cases} \text{VD}_{r_0}^{\varepsilon^{\text{MBC}}} & = \frac{q_{11}^2 b_{21}^2 + q_{21}^2 b_{22}^2 + 2q_{11}q_{21}b_{21}b_{22}}{b_{21}^2+b_{22}^2} \\ \text{VD}_{r_0}^{\varepsilon^{\text{SBC}}} & = \frac{q_{12}^2 b_{21}^2 + q_{22}^2 b_{22}^2 + 2q_{12}q_{22}b_{21}b_{22}}{b_{21}^2+b_{22}^2} \end{cases} \tag{41}$$

Suppose $b_{11} > 0$ & $b_{12} > 0$. Therefore, $b_{21} < 0$ & $b_{22} > 0$

Selecting q_{11} & q_{21} such that $\text{VD}_{y_0}^{\varepsilon^{\text{MBC}}}$ is maximized. This implies $q_{11} > 0$ & $q_{21} > 0$

Resulting in fall of $\text{VD}_{r_0}^{\varepsilon^{\text{MBC}}}$ as $b_{21}b_{22} < 0$

D Multiple Business Cycle Shocks

The following subsection shows that multiple shocks can be mapped to a single business cycle shock. Using the state-space representation and mapping to a vector of observables $\mathbf{Y}_t$:

$$S_t = \Theta_1(\theta)S_{t-1} + \Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t$$

$$Y_t = A(L)S_t = A(L)\left(I - \Theta_1(\theta)L\right)^{-1}\Theta_\epsilon(\theta)\Psi(\theta_1)\epsilon_t = \mathbf{D}(L;\theta)\mathbf{\Theta}_\epsilon(\theta)\mathbf{\Psi}(\theta_1)\epsilon_t$$

$$Y_t(k) = \mathbf{D}(L;\theta)\mathbf{\Theta}_\epsilon(\theta)\mathbf{\Psi}(\theta_1)\epsilon_t$$

$$Y_t(k, sB, lB) = \mathbf{D}^{\mathbf{k}}(L;\theta)\mathbf{\Theta}_\epsilon^{sB}\sigma_{sB}\epsilon_t^{sB} + \mathbf{D}^{\mathbf{k}}(L;\theta)\mathbf{\Theta}_\epsilon^{lB}\sigma_{lB}\epsilon_t^{lB}$$

Let $\sigma_{\mathbf{sB}} = \gamma\sigma_{\mathbf{lB}}$

$$Y_t(k, sB, lB) = \left(\mathbf{D}^{\mathbf{k}}(L;\theta)\mathbf{\Theta}_\epsilon^{sB}\gamma + \mathbf{D}^{\mathbf{k}}(L;\theta)\mathbf{\Theta}_\epsilon^{lB}\right)\sigma_{lB}\epsilon_t^{B}$$

$$Y_t(k, sB, lB) = \mathbf{D}^{\mathbf{k}}(L;\theta)\left(\mathbf{\Theta}_\epsilon^{sB}\gamma + \mathbf{\Theta}_\epsilon^{lB}\right)\sigma_{lB}\epsilon_t^{B}$$

Let $\Theta_\epsilon^B\sigma_B = \left(\mathbf{\Theta}_\epsilon^{sB}\gamma + \mathbf{\Theta}_\epsilon^{lB}\right)\sigma_{lB}$

The model implied Spectral Density of variable **k** in Y_t due to business cycle shock $\left(\varepsilon_t^B\right)$ at frequency ω is represented as:

$$\mathcal{SD}(\omega, k, \mathbf{B}; \theta, \theta_1) = \frac{1}{2\pi}\mathcal{M}(\omega, y_k, \mathbf{B}; \theta)\sigma_{\mathbf{B}}^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, \mathbf{B}; \theta) = \left|\mathbf{D}^k(e^{i\omega};\theta)\mathbf{\Theta}_\epsilon^{\mathbf{B}}(\theta)\right|^2$$

E Upward Bias

$$\begin{aligned}\mathcal{D}(y_k, \mathbf{B}, \mathbf{lNB}) &= \prod_{j=1}^{T}\left(\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \mathcal{D}(\omega_j, y_k, \mathbf{lNB})\kappa\right) \\ &\geq \prod_{j=1}^{T}\mathcal{D}(\omega_j, y_k, \mathbf{B}) + \prod_{j=1}^{T}\mathcal{D}(\omega_j, y_k, \mathbf{lNB})\kappa \quad = 1 + \kappa^T\end{aligned}$$

where for $\sigma_{\mathbf{lNB}}^2 > 0$,

$$\kappa = \frac{\sigma_{\mathbf{lNB}}^2}{\sigma_{\mathbf{B}}^2} > 0$$

F Log-linearized equations of the Smets and Wouter model (sticky-price-wage economy)

$$y_t = c_y c_t + i_y i_t + r^{kss} k_y \epsilon_t + \varepsilon_t^g \tag{42}$$

$$c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma} \mathbb{E}_t c_{t+1} + \frac{w^{ss} s^{ss} (\sigma_c - 1)}{c^{ss} \sigma_c (1+\lambda/\gamma)} (l_t - \mathbb{E}_t l_{t+1}) \tag{43}$$

$$- \frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c} (r_t - \mathbb{E}_t \pi_{t+1}) - \frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c} e_t^b \tag{44}$$

$$i_\ell = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{\ell-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathbb{E}_t i_{\ell+1} + \frac{1}{\Theta\gamma^2 (1+\beta\gamma^{(1-\sigma_c)})} q_t + \varepsilon_t^i \tag{45}$$

$$q_t = \beta(1-\delta)\gamma^{-\sigma_c} \mathbb{E}_t q_{t+1} - r_t + \mathbb{E}_t \pi_{t+1} + \left(1 - \beta(1-\delta)\gamma^{-\sigma_c}\right) \mathbb{E}_t r_{t+1}^k - \varepsilon_t^b \tag{46}$$

$$y_t = \Theta_p (\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a) \tag{47}$$

$$k_\ell^s = k_{t-1} + \epsilon_t \tag{48}$$

$$\epsilon_t = \frac{1-\psi}{\psi} r_t^k \tag{49}$$

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right) i_t + \left(1 - \frac{1-\delta}{\gamma}\right) \varphi\gamma^2 (1+\beta\gamma^{1-\sigma_c}) \varepsilon_\ell^i \tag{50}$$

$$\mu_t^p = \alpha(k_t^s - l_t) - w_t + \varepsilon_t^a \tag{51}$$

$$\pi_t = \frac{\beta\gamma^{1-\sigma_c}}{1+I_p\beta\gamma^{1-\sigma_c}} \mathbb{E}_t \pi_{t+1} + \frac{I_p}{1+\beta\gamma^{1-\sigma_c} I_p} \pi_{t-1} \tag{52}$$

$$- \frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{(1+I_p\beta\gamma^{1-\sigma_c})(1+(\Theta_p - 1)\varepsilon_p)\xi_p} \mu_t^p + \varepsilon_t^p \tag{53}$$

$$r_t^k = l_t + w_t - k_t \tag{54}$$

$$\mu_t^w = w_t - \sigma_l l_t - \frac{1}{1-\lambda/\gamma} (c_t - \frac{\lambda}{\gamma} c_{t-1}) \tag{55}$$

$$w_t = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} (\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) + \frac{1}{1+\beta\gamma^{1-\sigma_c}} (w_{t-1} + l_w \pi_{t-1}) \tag{56}$$

$$- \frac{1+\beta\gamma^{1-\sigma_c} I_w}{1+\beta\gamma^{1-\sigma_c}} \pi_t - \frac{(1-\beta\gamma^{1-\sigma_c}\xi_w)(1-\xi_w)}{(1+\beta\gamma^{1-\sigma_c})(1+(\varphi_w - 1)\varepsilon_w)\xi_w} \mu_t^w + \varepsilon_t^w \tag{57}$$

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y (y_t - y_t^*)) \tag{58}$$

$$+ r_{\Delta y}((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)) + \varepsilon_t^r \tag{59}$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{60}$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b \tag{61}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga} \eta_t^a + \eta_t^g \tag{62}$$

Note: The model variables are: output (y_t), consumption (c_t), investment (i_t), utilized and installed capital (k_t^s, k_t), capacity utilization (ϵ_t), rental rate of capital $\left(r_t^k\right)$, Tobin's q (q_t), price and wage markup (μ_t^p, μ_t^w), inflation rate (π_t), real wage (w_t), total hours worked (l_t), and nominal interest rate (r_t). The shocks are: total factor productivity (ε_t^a), investment-specific technology $\left(\varepsilon_t^i\right)$, government purchases (ε_t^g), risk premium $\left(\varepsilon_t^b\right)$, monetary policy (ε_t^r), wage markup (ε_t^w) and price markup (ε_t^p).

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