Dissecting Business Cycles

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Motivation

- **Goal: Relative role of long-run supply and short-run demand shocks in driving business cycles**
  - Identifying dynamic causal effects of business cycle shocks provides valuable insights into the internal propagation mechanism (amplification or persistence)
  - Monetary authority faces policy trade-offs due to long-run supply-driven business cycles
  - Literature has conflicting conclusions about the role of long-run supply-driven business cycles

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  - Literature has conflicting conclusions about the role of long-run supply-driven business cycles

- **Literature**: Identifies **long-run productivity** shocks $\xRightarrow{\text{evaluate}}$ **business-cycle GDP** fluctuations

- **This Paper**: Dissects GDP fluctuations to identify shocks that explain business-cycle volatility of GDP

  Identified **business-cycle shocks** $\xRightarrow{\text{evaluate}}$ **long-run productivity** fluctuations

But why a new approach?

Allows for **two** categories of long-run productivity shocks. One causes business cycles and the other doesn't.

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  Q: Does an average aggregate long-run TFP shock drive business cycles?

  * **Two assumptions**:
    1. Long-run TFP shocks are exogenous
    2. There exists only one category of long-run productivity shock

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  Q: Does there exist any subset of long-run TFP shocks that may drive business cycles?

  - Weakens assumption **2**. Allows for two categories of long-run productivity shocks
  - Assumption **1** holds. Avoids reverse causality

Business Cycle Shocks

- **ACD: Angeletos, Collard & Dellas (2020)**:
  - Argue non-inflationary demand shocks drive business cycles. MBC
  - Extract a shock that explains maximum business cycle volatility of real per capita GDP.
- **Key Assumption**: Business cycles have a dynamic factor structure and there's one factor.
  - In other words, single shock drives business cycles.
  - **MBC** shock: 1st principal component

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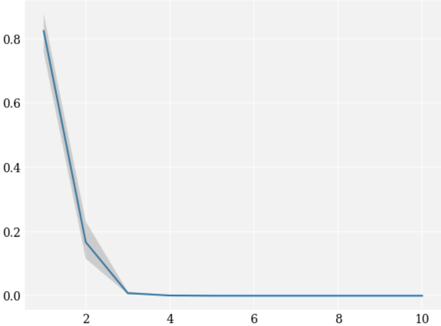
  - In other words, single shock drives business cycles.
  - **MBC** shock: 1st principal component
- I test this key assumption on the number of dynamic factors.
  - There are two factors.
  - Separate them using a hypothesis, some of these shocks have long-run implications and some don't.
  - Based on empirical results, I interpret the two shocks as supply and demand shocks.

Number of Dynamic Factors?

Figure Scree Plot



Eigenvalues for a spectral matrix of GDP at business cycle frequency band.
Horizontal axis: Total principal components or eigenvalues.

- **This Paper**: The **MBC** shock is a linear combination of supply and demand shocks

Overview: Results

Using a novel SVAR identification strategy to dissect business cycle fluctuations:

- **Yes**, a significant fraction of long-run TFP shocks drive business cycles
- **Sources of Business Cycle Fluctuations**:
  - Identify two business cycle shocks, a short-run and a long-run shock
  - Further identified as a long-run **supply shock** and a short-run **demand shock** based on conditional correlations of macro variables

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  - Further identified as a long-run **supply shock** and a short-run **demand shock** based on conditional correlations of macro variables
- Also a **second** category of long-run TFP shocks that don't drive business cycles
  - Leads to biased parameters of DSGE models estimated in a full information setting
  - Significant **normative** & **policy** implications
  - **Solution**: Estimation via IRF matching with the identified business cycle shocks

Literature Review

- **SVAR Identification (Technology Shocks)**:

  Blanchard & Quah (1989); Gali (1999); Basu, Fernald & Kimball (2006); Beaudry & Portier (2006); Barsky & Sims (2011); Francis et al. (2014); Barsky, Basu & Lee (2014); Chahrour & Jurado (2018); Angeletos, Collard & Dellas (2020); Kurmann & Sims (2022); Chahrour, Chugh & Potter (2022);

  - Conflicting conclusions about the role of long-run TFP shocks
  - Argue for (non-inflationary) demand shocks as the key driver of business cycles.

- **Limited Information Estimation**:

  Rotemberg & Woodford (1997), Christiano, Eichenbaum & Evans (2005), Barnichon & Mesters (2020), Lewis & Mertens (2023)

  - Identify macro equations through structural shocks

Outline

1. Identification Setup
2. Results
3. Model Estimation Challenges
4. Application: Smets & Wouters (2007)

Empirical Analysis

Baseline VAR

- Data follows the benchmark VAR of ACD (2020):
  - Quarterly U.S data: 1955Q1-2019Q4
  - **Macro Quantities**: Unemployment, GDP, Hours, Invest. (inclusive of durables), Cons.
  - **Productivity**: util-adjust TFP, NFB labor productivity;
  - **Nominal**: Inflation (GDP Delator), Federal Fund Rate, Labor Share
  - Bayesian VAR, 2 Lags
- **Wold Representation**:

$$Y_t = D(L)Q\varepsilon_t$$

where, ε_t are structural shocks.

Identification

BQ 1989

$$\varepsilon_t' = [\ \underbrace{\varepsilon_{B,t}^{long-run} \qquad \varepsilon_{B,t}^{short-run}}_{\text{Business cycle shocks}} \qquad \underbrace{\varepsilon_{NB,t}}_{\text{Non-Business Cycle shocks}}\ ]$$

- **B**: Linear combination of the VAR residuals that explain significant volatility of **GDP** at the business-cycle frequencies, 6-32 quarters
- $\varepsilon_{B,t}^{short-run}$: Business cycle shocks that don't contribute to long-run volatility of GDP
- Following ACD (2020), **long-run** refers to fluctuations of periodicity >20 years
- $\varepsilon_{B,t}^{long-run}$: Residual business cycle shocks
- Structural assumptions consistent with the literature.

Identification Setup

Definition BQ 1989

$$q_{lr}^*, q_{sr}^* \equiv \arg\max_{q_{lr}, q_{sr}} \; q_{lr}{}'\mathcal{D}\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{lr} + q_{sr}{}'\mathcal{D}\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{sr}$$
$$- q_{sr}{}'\mathcal{D}\left(GDP, \frac{2\pi}{\infty}, \frac{2\pi}{80}\right) q_{sr}$$
$$\text{s.t. } q_{lr}' q_{lr} = 1, q_{sr}' q_{sr} = 1, q_{lr}' q_{sr} = 0$$

- Identify two orthogonal shocks q_{lr}^* and q_{sr}^*

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- Identify two orthogonal shocks q^*_{lr} and q^*_{sr}
- Both together explain the maximum volatility of real per capita GDP at business cycle frequency
- Penalize q^*_{sr} for explaining long-run volatility of GDP
- Results robust to long-run restrictions via labor productivity, TFP, Consumption

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- Identify two orthogonal shocks q_{lr}^* and q_{sr}^*
- Both together explain the maximum volatility of real per capita GDP at business cycle frequency
- Penalize q_{sr}^* for explaining long-run volatility of GDP
- Results robust to long-run restrictions via labor productivity, TFP, Consumption
- Key: Not rewarding q_{lr}^* for explaining long-run TFP movements

Identification Setup

- **MBC** shock (q_1): principal component analysis

$$\max_{q_1,q_2} q_1' A q_1 + q_2' A q_2$$

$$\text{s.t. } q_1' q_1 = 1, q_2' q_1 = 1, q_2' q_1 = 0$$

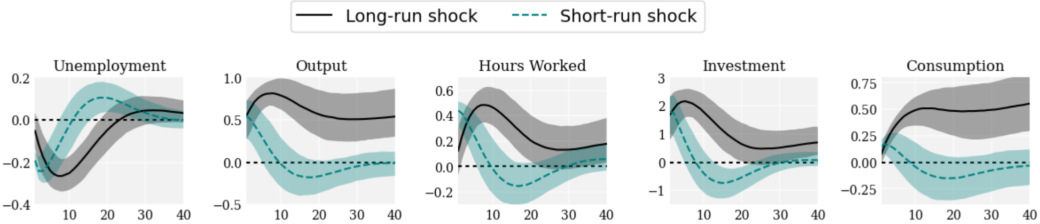
- **This paper:** extrema of sums of heterogeneous quadratic forms (A $\neq$ B)

$$\max_{q_1,q_2} q_1' A q_1 + q_2' B q_2$$

$$\text{s.t. } q_1' q_1 = 1, q_2' q_1 = 1, q_2' q_1 = 0$$

- **Existence & Uniqueness**: Bolla, M., Michaletzky, G., Tusnády, G., Ziermann, M. (1998)
- **Convergence Algorithm**: Jiang & Dai (2014)

Business Cycle Co-movement



- Volatility contribution at business-cycle frequency band (6-32 quarters):

Shock	Unemployment	Output	Hours Work	Investment	Consumption
Long-run	33.9	56.6	30.8	43.8	32.8
	[22.8, 46.4]	[36.1, 73.7]	[22, 41.3]	[27.4, 59.1]	[25.7, 40.2]
Short-run	46.8	42.1	39.4	41.3	23
	[34.1, 57.7]	[25.2, 63]	[28.8, 48.3]	[25.6, 57.7]	[16.1, 30.8]
Total	80.7	98.7	70.2	85.1	55.8

NOTE. 80 percent HPDI in brackets

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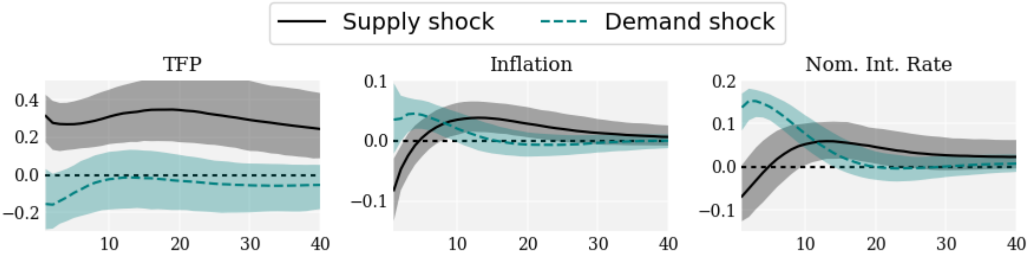
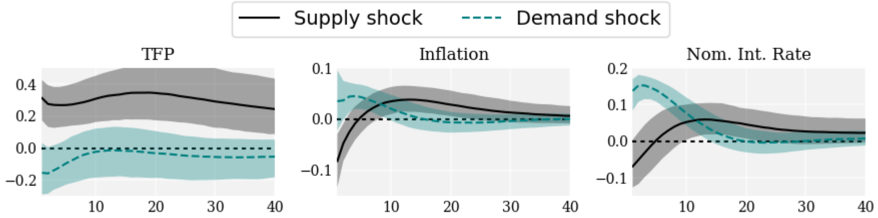
TFP, Inflation & Interest Rates



- **Supply shock** (TFP) $\uparrow \implies$ GDP $\uparrow \implies$ inflation $\downarrow \xrightarrow{\text{Taylor Rule}}$ nominal rates $\downarrow$
- **Demand shock** $\uparrow \implies$ GDP $\uparrow \implies$ inflation $\uparrow \xrightarrow{\text{Taylor Rule}}$ nominal rates $\uparrow$

TFP, Inflation & Interest Rates



	TFP (6-32 Q)	TFP (> 80Q)	Inflation	Nominal Int Rates
Supply Shock (long-run)	12.7	53.1	18.2	17.6
	[5.9, 22.5]	[28.7, 71.3]	[10.4, 28.4]	[8.4, 33.1]
Demand Shock (short-run)	8.3	0.23	11.7	52.5
	[3, 17.3]	[0.03, 1.05]	[5.8, 19.8]	[39.2, 62.1]

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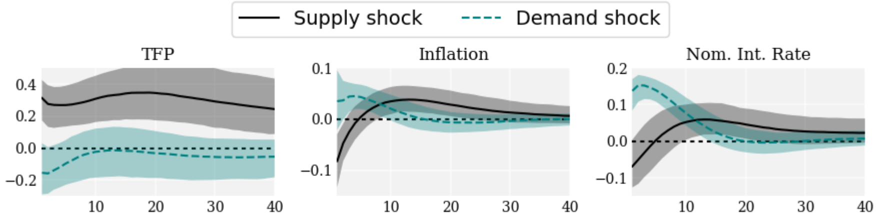

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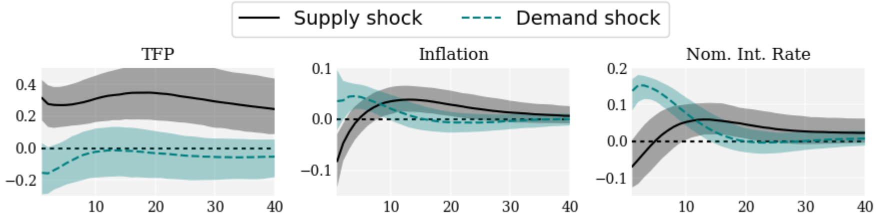

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- The **MBC** shock is a linear combination of long-run supply and short-run demand shocks
- Also evidence for significant long-run TFP shocks that don't drive business cycles

Limited Information Estimation

Smets & Wouters (2007)

- Using a **Bayesian likelihood** approach, estimate a medium-scale **DSGE** model to investigate:
  - Relative empirical importance of the various frictions
  - Sources of business cycle fluctuations
  - Policy analysis
- **Components**:
  1. Adjustment costs for investment
  2. Capacity utilization costs
  3. Habit persistence
  4. Price & wage indexation and nominal rigidities
  5. Seven structural shocks (one long-run, six short-run)
- **Seven Observables**: GDP, Consumption, Investment, Wages, Hours Worked, Inflation, FFR

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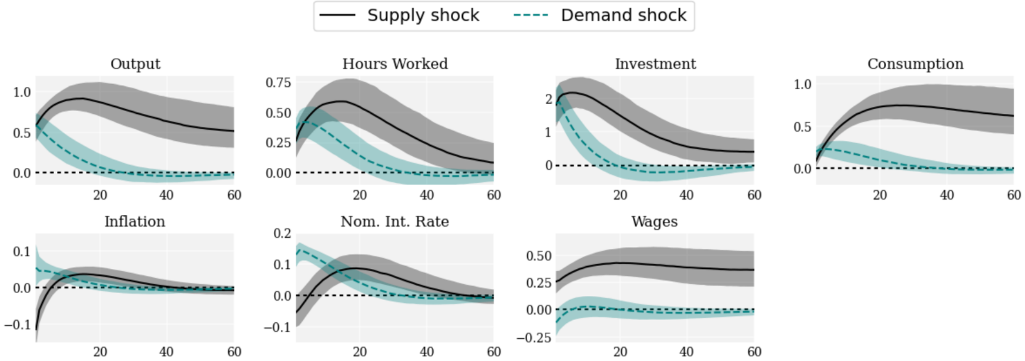
- **This Paper**: Estimates parameters via IRF matching

Dissecting Smets-Wouters Observables



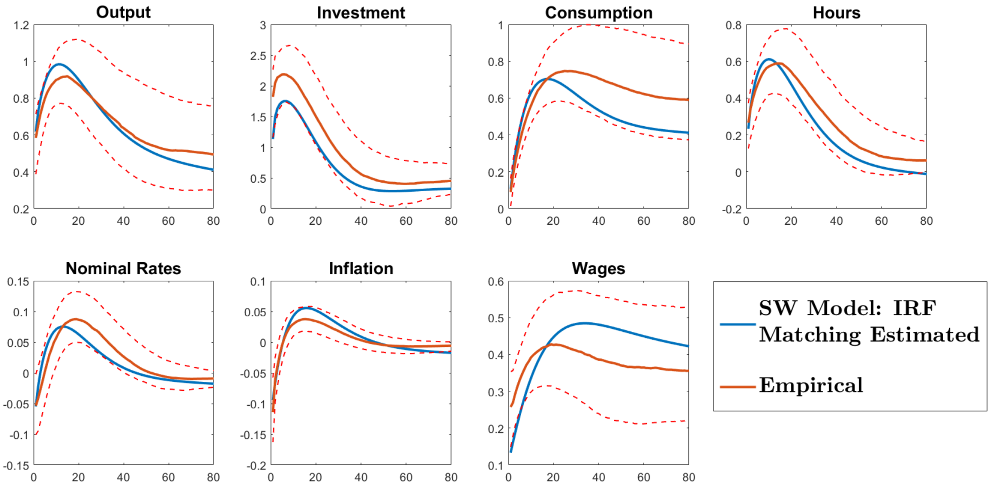
- Conclusions from empirical analysis section hold

Volatility Contributions

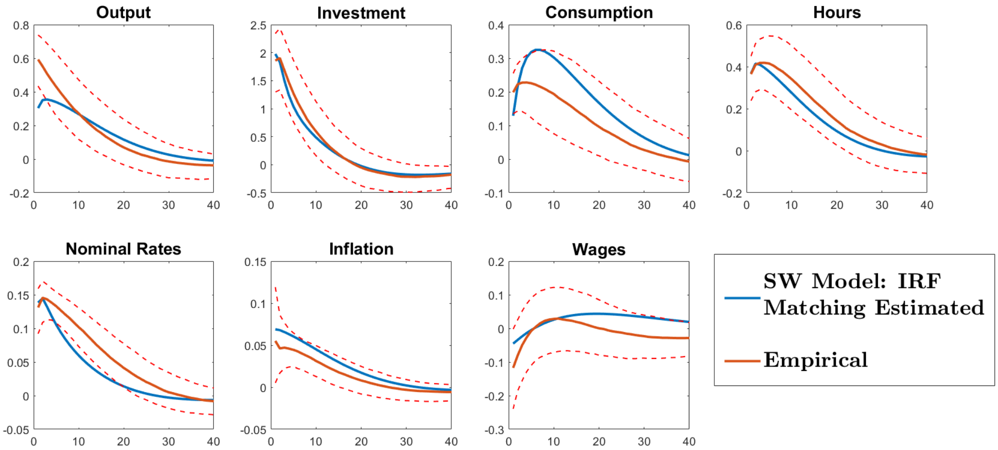
Shock	Output	Hours Work	Investment	Consumption
Supply (Business Cycle Volatility)	57.3	23.8	43	26.8
	[33.9, 76]	[10.6, 36.1]	[22.7, 60.3]	[18.7, 36.6]
Demand (Business Cycle Volatility)	42.5	30.7	38.4	17.9
	[23.6, 65.7]	[16, 45.6]	[20.4, 59.8]	[8.3, 27.8]
Total (Business Cycle Volatility)	99.8	54.5	81.7	44.7
Supply (Long-run Volatility)	66.5	69.9	69.11	65
	[36.9, 86.4]	[44.8, 84.4]	[40, 86.5]	[34.9, 85.3]

Shock	Inflation	FFR	Wages
Supply (Business Cycle Volatility)	22.3	15.3	19.8
	[10.3, 37.9]	[6.3, 30.9]	[10.5, 32.5]
Demand (Business Cycle Volatility)	8.3	41.2	5.4
	[3.1, 21]	[25.4, 53.6]	[2.3, 12.7]
Total (Business Cycle Volatility)	30.6	56.5	25.2
Supply (Long-run Volatility)	27.7	27.7	65.3
	[11, 56]	[13, 51.7]	[33.8, 85.2]

Supply Shock: SW Long-run TFP Shock



Demand Shock: SW Risk Premia Shock



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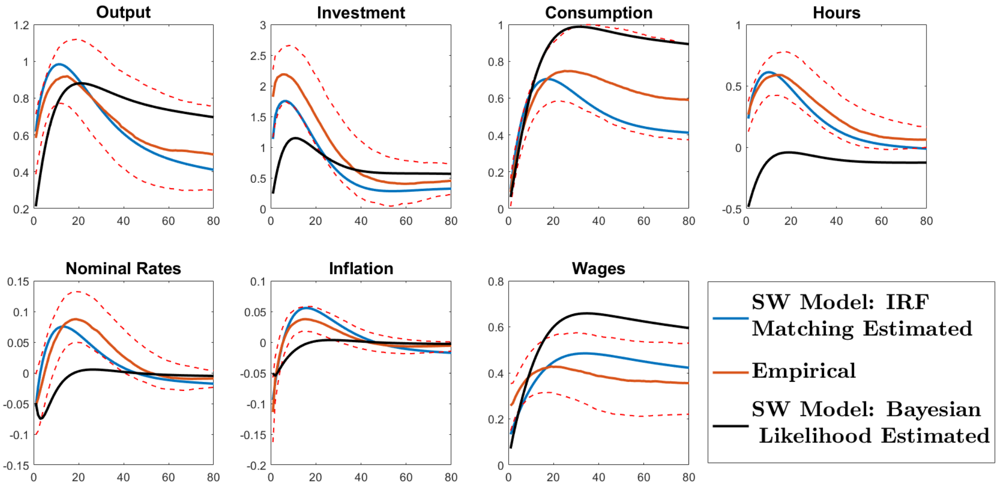
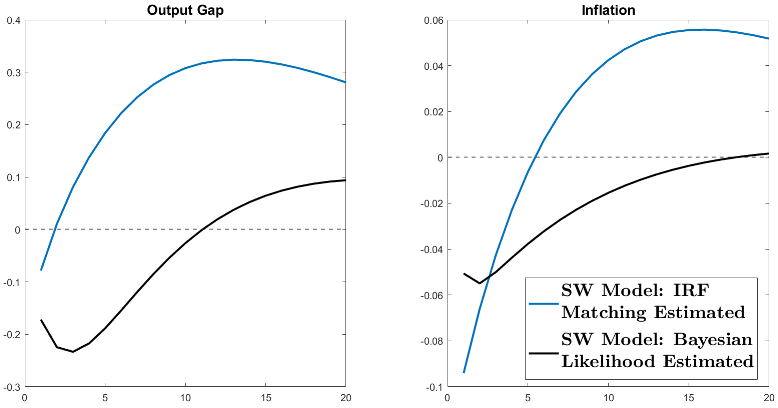


Normative & Policy Implications



- Policy trade-offs in IRF matching estimated model.

Estimation Challenges

Full Information Estimation Challenges

In the following section,

- I argue for downward bias in business cycle implications of **DSGE** models estimated using Bayesian likelihood:

    1. DSGE models have cross-frequency restrictions
    2. Presence of long-run Non-Business Cycle shocks result in downward bias

Spectral Representation of DSGE Model

- Canonical representation of the DSGE model:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Psi Z_t + \Pi \zeta_t$$

$\mathbf{S}_t$: Endogenous Variables, $\mathbf{Z}_t$: Exogenous Shocks, ζ_t: Expectational shocks

- Assuming a state-space representation and maping to observables $\mathbf{Y}_t$:

$$S_t = \Theta_1 S_{t-1} + \Theta_0 \Psi Z_t$$

$$Y_t = A(L) S_t = A(L) \left(I - \Theta_1 L \right)^{-1} \Theta_0 \Psi Z_t = \mathbf{D}(L; \theta) \mathbf{\Theta}_0(\theta) \mathbf{\Psi}(\theta_1) Z_t$$

θ: model parameters, θ_1: shock standard deviations

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- Model implied **Spectral Density** of variable **k** due to shock **l** in $\mathbf{Y}_t$:

$$\mathcal{SD}(\omega, k, l; \theta, \theta_1) = \frac{1}{2\pi} \left|\mathcal{M}(\omega, y_k, l; \theta)\right|^2 \sigma_l^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, l; \theta) = \mathbf{D}^k(e^{i\omega}; \theta) \mathbf{\Theta}_0^l(\theta)$$

Likelihood Function of DSGE Models

Application

- The log-likelihood function of the state space model in frequency domain (Harvey 1989)

$$\log L\left(\theta,\theta_1\right) = -\sum_{j=1}^{T}\left(\log \frac{1}{2\pi}\left|\mathcal{M}(\omega_j, y_k, lB;\theta)\right|^2 \sigma_{lB}^2 + \frac{I(\omega_j, y_k)}{\frac{1}{2\pi}\left|\mathcal{M}(\omega_j, y_k, lB;\theta)\right|^2 \sigma_{lB}^2}\right)$$

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- Maximising log L with respect to σ_{lB}^2 gives:

$$\tilde{\sigma}_{lB}^2(\theta) = \frac{2\pi}{T}\sum_{j=1}^{T}\frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2} = \frac{2\pi}{T}S(\theta)$$

- Reducing the maximize log L objective to **minimising**

$$S(\theta) = \underbrace{\sum_{j=1}^{t}\frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{long-run volatility}} + \underbrace{\sum_{j=t+1}^{T}\frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{short-run volatility}}$$

Cross-Frequency Restrictions

- Simplifying objective function into long-run and short-run volatility:

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- **Data implied volatility**:

$$I\left(\omega_j, k\right) = \frac{1}{2\pi}\mathcal{D}(y_k, \omega_j, lB)\sigma_{lB}^2 + \frac{1}{2\pi}\mathcal{D}(y_k, \omega_j, lNB)\sigma_{lNB}^2$$

- **Cross-frequency Restriction**: Kolmogorov result

$$\underbrace{\sum_{j=1}^{t} \log\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}_{\text{long-run}} + \underbrace{\sum_{j=t+1}^{T} \log\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}_{\text{short-run}} = 0$$

Cross-Frequency Restrictions

- Suppose $\exists\ \theta^*$ s.t. $\mathcal{D}(y_k, \omega_j, lB) = \left|\mathcal{M}(\omega_j, y_k, lB; \theta^*)\right|^2\ \forall\ \omega_j$

$$S(\theta) = \underbrace{\sum_{j=1}^{t} \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, lB)\sigma_{lB}^2 + \mathcal{D}(y_k, \omega_j, lNB)\sigma_{lNB}^2}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{long-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, lB)\sigma_{lB}^2 + 0}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{short-run volatility}}$$

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$$S(\theta^*) = \underbrace{\sum_{j=1}^{t} \frac{\sigma^2_{lB}}{2\pi} + \frac{\mathcal{D}(y_k, \omega_j, lNB)\sigma^2_{lNB}}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta^*)\right|^2}}_{\text{long-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{\sigma^2_{lB}}{2\pi}}_{\text{short-run volatility}} = T\frac{\sigma^2_{lB}}{2\pi} + \sum_{j=1}^{t} \frac{\mathcal{D}(y_k, \omega_j, lNB)\sigma^2_{lNB}}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta^*)\right|^2}$$

- Minimizes $S(\theta)$ to $T\frac{\sigma^2_{lB}}{2\pi}$ for true parameter (θ^*) values if $\sigma^2_{lNB} = 0$

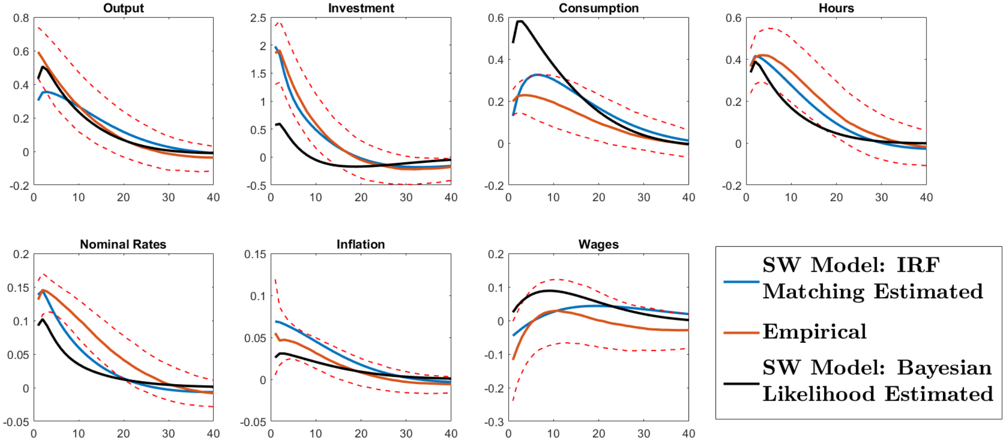
Downward Bias for Business Cycles

$$S(\theta) = \underbrace{\sum_{j=1}^{t} \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, lB)\sigma_{lB}^2 + \mathcal{D}(y_k, \omega_j, lNB)\sigma_{lNB}^2}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{long-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, lB)\sigma_{lB}^2 + 0}{\left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}}_{\text{short-run volatility}}$$

$$\sigma_{lNB}^2 \uparrow \xRightarrow{minimize \quad S(\theta)} \underbrace{\sum_{j=1}^{t} \left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}_{\text{long-run}} \uparrow \xRightarrow{restriction} \underbrace{\sum_{j=t+1}^{T} \left|\mathcal{M}(\omega_j, y_k, lB; \theta)\right|^2}_{\text{short-run}} \downarrow$$

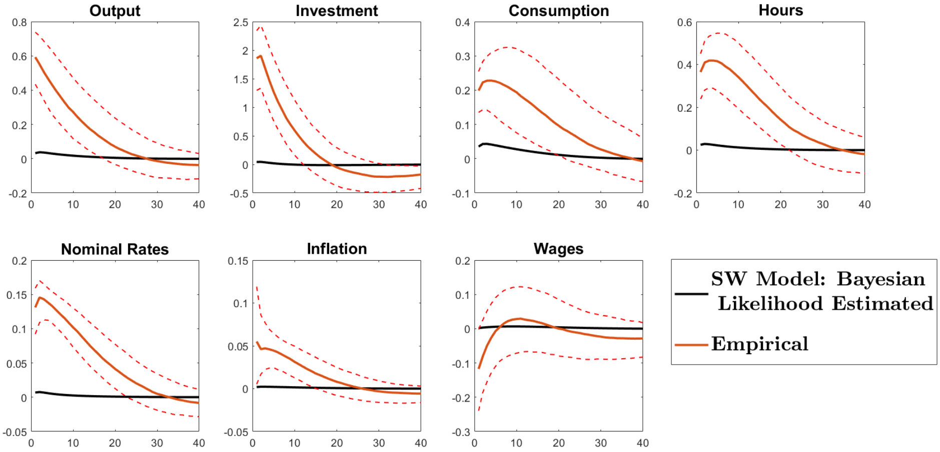
- θ changes such that model implied long-run volatility increases, resulting in a **downward bias** on short-run volatility of the model
- Argues for estimation in a limited information setting

Demand Shock: SW Risk Premia Shock



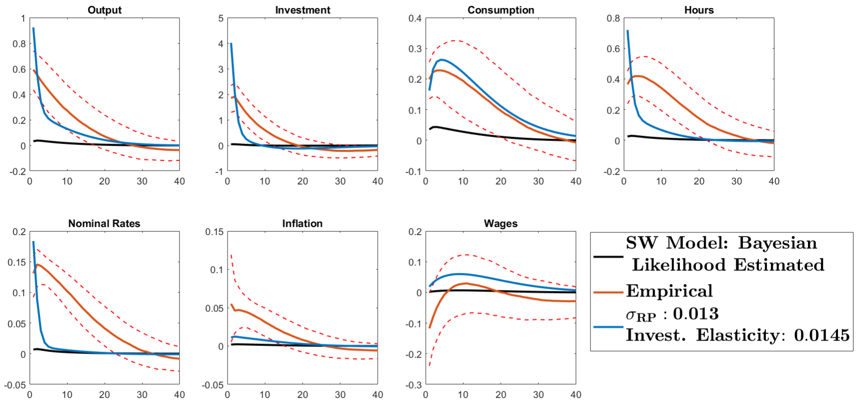
Internal vs. External Propagation

Theory



- Replaced likelihood estimated σ_{RP} (0.1762) with IRF matched estimation (0.0131)

Internal vs. External Propagation



- Additionally replaced likelihood estimated investment elasticity (8.0145) with IRF matching estimated (0.0145)

Conclusion

- **Empirical Results**:
  1. Both long-run supply and short-run demand shocks drive business cycles
  2. DGP also comprises long-run shocks that don't contribute to business cycles
- **Estimation Results**:
  1. Long-run non-business cycle shocks result in a downward bias in business cycle implications of DSGE models estimated in full-information setting
  2. **Solution**: Estimation in limited information setting
     - For instance, estimation of Smets & Wouters (2007) by IRF matching with the identified shocks.

Appendix Slides

Representation

- Wold Representation:

$$Y_t = D(L)Q\varepsilon_t$$

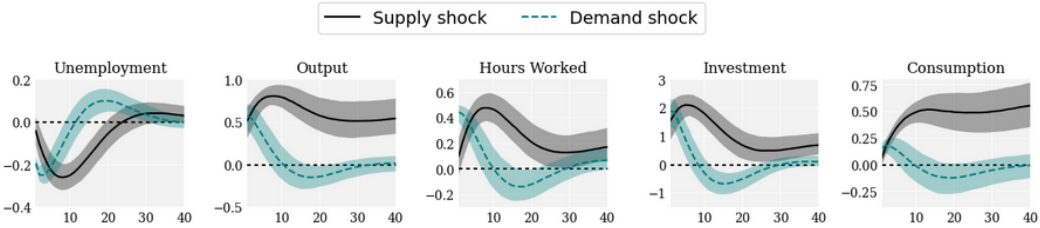
- Spectral density of a variable y_j in Y_t in the frequency band $[\underline{\omega}, \bar{\omega}]$ is represented as:

$$\mathcal{D}(y_j, \underline{\omega}, \bar{\omega}) = \int_{\underline{\omega}}^{\bar{\omega}} \left(\overline{D^j\left(e^{-i\omega}\right)} D^j\left(e^{-i\omega}\right) \right) d\omega$$

- For instance, spectral density of GDP in business-cycle frequency band (6-32 quarters):

$$\mathcal{D}\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right)$$

Business Cycle Co-movement



► Volatility contribution at business-cycle frequency band (6-32 quarters):

	Unemployment	Output	hours Work	Investment	Consumption
Supply Shock	31.1	49.5	30	38.8	32.5
	[20.7, 45.8]	[30.5, 71.6]	[20.5, 40.7]	[23.4, 58]	[24.7, 39.9]
Demand Shock	49.5	49	40.7	45.8	23.4
	[35.4, 59.4]	[26.3, 67.8]	[28, 49.2]	[26.4, 61.6]	[15.7, 31.7]

NOTE. 68 percent HPDI in brackets

Check: 1

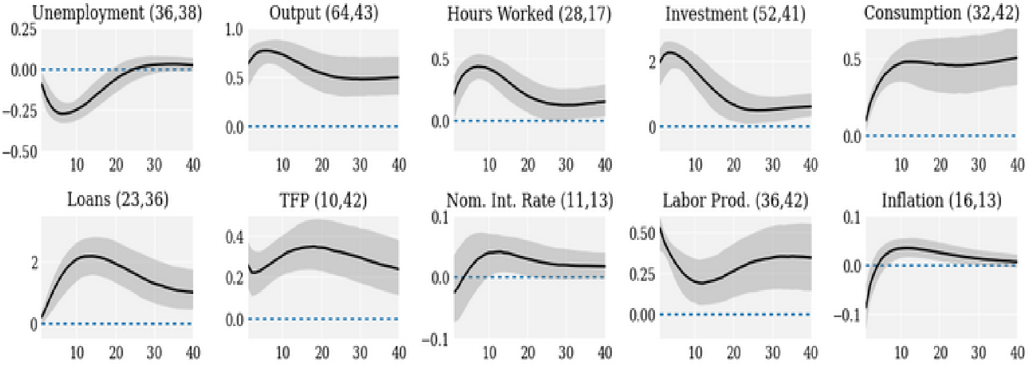
$$q_{lr}, q_{sr} \equiv \arg\max_{q_{lr}, q_{sr}} {q_{lr}}' D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{lr} +$$

$$ {q_{sr}}' \left(1.01 \;\; D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) - D\left(TFP, \frac{2\pi}{\infty}, \frac{2\pi}{80}\right)\right) q_{sr}$$

$$\text{s.t. } q'_{lr} q_{lr} = 1, q'_{sr} q_{sr} = 1, q'_{lr} q_{sr} = 0$$

Supply Shock IRFs

Figure IRFs



Check: 2

TFP

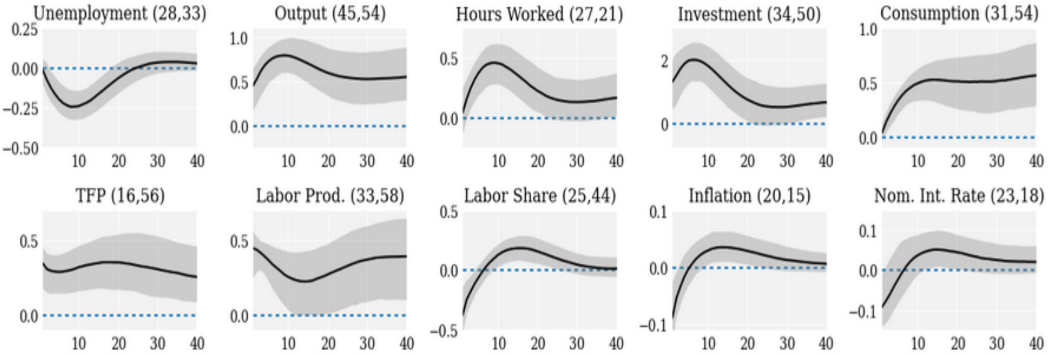
$$q_{lr}, q_{sr} \equiv \arg\max_{q_{lr}, q_{sr}} q_{lr}{}' D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{lr} +$$

$$q_{sr}{}' \left(1.1 \;\; D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) - D\left(TFP, \frac{2\pi}{\infty}, \frac{2\pi}{80}\right)\right) q_{sr}$$

$$\text{s.t. } q'_{lr} q_{lr} = 1, q'_{sr} q_{sr} = 1, q'_{lr} q_{sr} = 0$$

Supply Shock IRFs

Figure IRFs



Blanchard & Quah (1989)

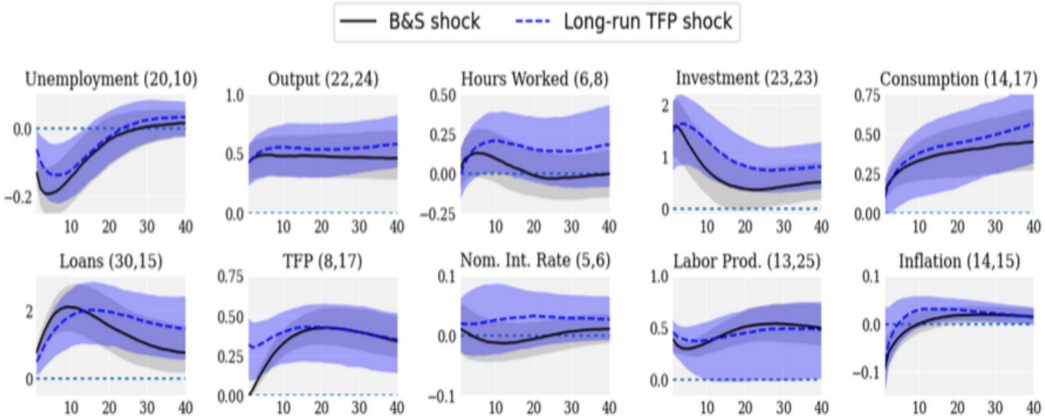
- **Blanchard & Quah (1989)**:
  - A bivariate VAR analysis of real GDP and unemployment.
  - Zero long-run restriction: Only the aggregate supply shock has permanent effects on the level of real GDP.
  - The residual orthogonal shock is interpreted as an aggregate demand shock.
  - They argue aggregate demand shocks as a key driver of business cycles.

$$\varepsilon_t' = [\ \underbrace{\varepsilon_{B,t}^{long-run'} \quad \varepsilon_{B,t}^{short-run} \quad \varepsilon_{NB,t}^{residual}}_{\text{Aggregate Demand shock}} \quad \underbrace{\varepsilon_{B,t}^{perm} \quad \varepsilon_{NB,t}^{perm}}_{\text{Aggregate Supply shock}}\ ]$$

  - Confounds business and non-business cycle shocks.

Barsky & Sims (2011) vs. Long-run TFP Shocks

BS vs KS



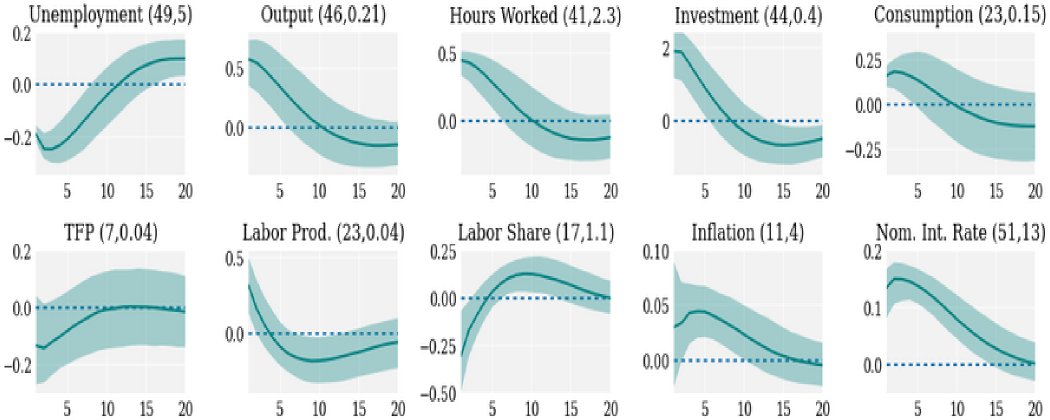
- Long-run TFP shocks from Angeletos, Collard & Dellas (2020)
- Similar IRFs and business cycle volatility for macro variables.

Overview: Model Estimation Results

In light of the evidence where we have two categories of long-run TFP shocks:

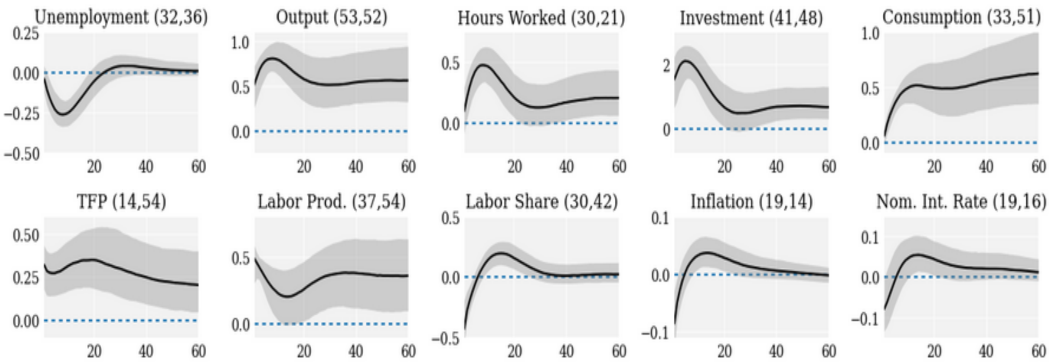
- Benchmark medium-scale **DSGE** models have **model misspecification**
  - Similar to SVAR literature, DSGE models allow for one category of long-run TFP shocks For instance, Smets & Wouters (2007)
  - Full-information likelihood-based estimation of these models results in biased parameters
  - Downward bias on business cycle implications of such models.
- **Solution**: Estimation in limited information setting
  - Estimation of Smets & Wouters (2007) by IRF matching with the identified shocks.
- **Result**: Wage indexation and stickiness are key for propagation mechanism relative to price & investment frictions.

Demand Shock (short-run)



NOTE. 80 percent HPDI in brackets

Supply Shock (long-run)



- Explains significant volatility of Une, Y, h, I and C at both frequency bands.
- Explains only long-run fluctuations of TFP.
- Explains significant labor productivity (Y/h) fluctuations at both frequency bands.

Output Periodogram

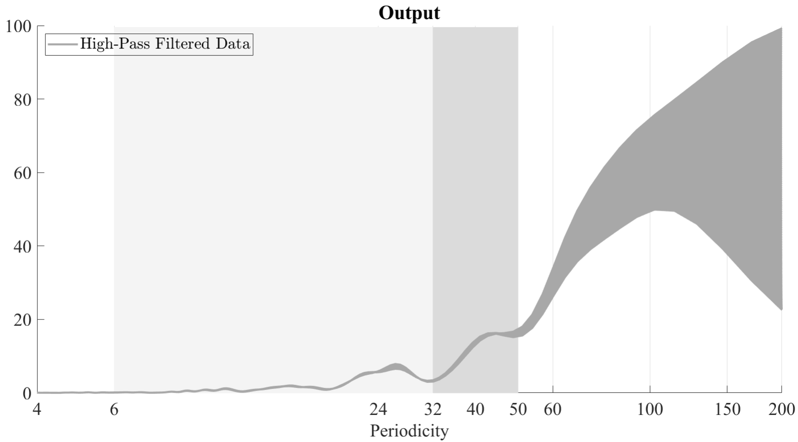
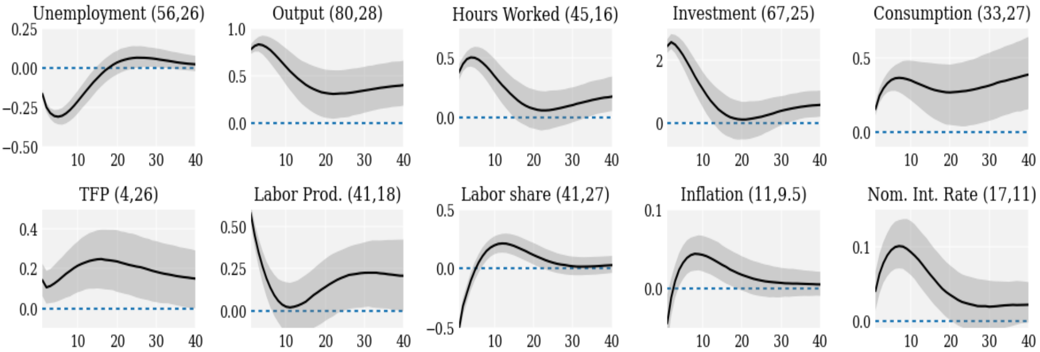


Figure This figure shows an estimate of the spectral density of U.S. GDP per capita filtered for periodicity above 20 quarters.

ACD (2020): MBC Shock

Figure IRFs



Normative & Policy Implications