Dissecting Business Cycles

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Motivation

 Goal: Relative role of long-run supply and short-run demand shocks in driving business cycles

- * Monetary authority faces policy trade-offs due to long-run supply-driven business cycles
- * SVAR literature has conflicting conclusions about the causal effects of long-run supply shocks
- * **DSGE** literature has conflicting conclusions about the normative and policy implications of long-run supply shocks

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- * **DSGE** literature has conflicting conclusions about the normative and policy implications of long-run supply shocks
- ► Literature: Identifies long-run productivity shocks → business-cycle GDP fluctuations
- This Paper: Dissects GDP fluctuations to identify shocks that explain business-cycle volatility of GDP

Identified **business-cycle shocks** long-run productivity fluctuations

But why a new approach?

Allows for **two** categories of long-run productivity shocks. One causes business cycles and the other doesn't.

Literature: Identifies long-run productivity shocks business-cycle GDP fluctuations

Q: Does an average aggregate long-run TFP shock drive business cycles?

- * Two assumptions:
 - 1. Long-run TFP shocks are exogenous
 - 2. There exists only one category of long-run productivity shock
- ► This Paper: Identifies business-cycle shocks evaluate long-run productivity fluctuations

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- ► This Paper: Identifies business-cycle shocks evaluate long-run productivity fluctuations

Q: Does there exist any subset of long-run TFP shocks that may drive business cycles?

- * Relaxes assumption 2. Allows for two categories of long-run productivity shocks
- * Assumption 1 holds. Avoids reverse causality

Business Cycle Shocks

- ACD: Angeletos, Collard & Dellas (2020):
 - * Argue non-inflationary demand shocks drive business cycles.
 - * Extract a shock that explains maximum business cycle volatility of real per capita GDP.
- **Key Assumption**: Business cycles have a dynamic factor structure and there's one factor.
 - * In other words, single shock drives business cycles.
 - * MBC shock: 1st principal component

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- **Key Assumption**: Business cycles have a dynamic factor structure and there's one factor.
 - * In other words, single shock drives business cycles.
 - * MBC shock: 1st principal component
- I test this key assumption on the number of dynamic factors.
 - * There are two factors.
 - * Separate them using a hypothesis, some of these shocks have long-run implications and some don't.
 - * Based on empirical results, I interpret the two shocks as supply and demand shocks.

Number of Dynamic Factors?



> This Paper: The MBC shock is a linear combination of supply and demand shocks

Overview: Results

Using a novel SVAR identification strategy to dissect business cycle fluctuations:

> Yes, a significant fraction of long-run TFP shocks drive business cycles

Identification of Dynamic Causal Effects

- * Identify two business cycle shocks, a short-run and a long-run shock
- * Further identified as a long-run **supply shock** and a short-run **demand shock** based on conditional correlations of macro variables
- * A **second** category of long-run shocks that don't drive business cycles

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Identification of Model Parameters

- * Theoretically argue non-business cycle fluctuations lead to biased parameters of DSGE models estimated in a full information setting
- * Significant normative & policy implications
- * Solution: Estimation via IRF matching with the identified business cycle shocks

Literature Review

SVAR Identification (TFP Shocks):

Blanchard & Quah (1989); Gali (1999); Basu, Fernald & Kimball (2006); Beaudry & Portier (2006); Barsky & Sims (2011); Francis et al. (2014); Barsky, Basu & Lee (2014); Chahrour & Jurado (2018); Angeletos, Collard & Dellas (2020); Kurmann & Sims (2022); Chahrour, Chugh & Potter (2022);

- * Conflicting conclusions about the role of long-run TFP shocks
- * **Contribution**: Relaxes the common assumption about one category of long-productivity shock

Limited Information Estimation:

Rotemberg & Woodford (1997), Christiano, Eichenbaum & Evans (2005), Barnichon & Mesters (2020), Lewis & Mertens (2023)

* Contribution: Argues for limited information estimation due to non-business cycle fluctuations

Outline

- 1. Identification Setup
- 2. Results
- 3. Model Estimation Challenges
- 4. Application: Smets & Wouters (2007)

Empirical Analysis

Baseline VAR

Data follows the benchmark VAR of ACD (2020):

- * Quarterly U.S data: 1955Q1-2019Q4
- * Macro Quantities: Unemployment, GDP, Hours, Invest. (inclusive of durables), Cons.
- * Productivity: util-adjust TFP, NFB labor productivity;
- * Nominal: Inflation (GDP Delator), Federal Fund Rate, Labor Share
- * Bayesian VAR, 2 Lags

Wold Representation:

 $Y_t = D(L)Q\varepsilon_t$

where, ε_t are structural shocks.

Identification





- B: Linear combination of the VAR residuals that explain significant volatility of GDP at the business-cycle frequencies, 6-32 quarters
- $\triangleright \varepsilon_{B,t}^{short-run}$: Business cycle shocks that don't contribute to long-run volatility of GDP
- ▶ Following ACD (2020), long-run refers to fluctuations of periodicity >20 years
- $\triangleright \varepsilon_{B,t}^{long-run}$: Residual business cycle shocks
- Structural assumptions consistent with the literature.

$$q_{lr}^{*}, q_{sr}^{*} \equiv \arg \max_{q_{lr}, q_{sr}} q_{lr}' \mathcal{D} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{lr} + q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{sr} - q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{\infty}, \frac{2\pi}{80} \right) q_{sr}$$

s.t. $q_{lr}' q_{lr} = 1, q_{sr}' q_{sr} = 1, q_{lr}' q_{sr} = 0$

• Identify two orthogonal shocks q_{lr}^* and q_{sr}^*

Definition

BQ 1989

HOF

(

$$q_{lr}^{*}, q_{sr}^{*} \equiv \arg \max_{q_{lr}, q_{sr}} \frac{q_{lr}}{\mathcal{D}} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{lr} + q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{sr} - q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{\infty}, \frac{2\pi}{80} \right) q_{sr}$$

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- Both together explain the maximum volatility of real per capita GDP at business cycle frequency

BO 1989

HOF

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- Both together explain the maximum volatility of real per capita GDP at business cycle frequency
- Penalize q_{sr}^* for explaining long-run volatility of GDP

BO 108

$$q_{lr}^{*}, q_{sr}^{*} \equiv \arg \max_{q_{lr}, q_{sr}} \frac{q_{lr}}{D} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{lr} + q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6} \right) q_{sr} - q_{sr}' \mathcal{D} \left(GDP, \frac{2\pi}{\infty}, \frac{2\pi}{80} \right) q_{sr}$$

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- Results robust to long-run restrictions via labor productivity, TFP, Consumption

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- Identify two orthogonal shocks q_{lr}^* and q_{sr}^*
- Both together explain the maximum volatility of real per capita GDP at business cycle frequency
- Penalize q_{sr}^* for explaining long-run volatility of GDP
- Results robust to long-run restrictions via labor productivity, TFP, Consumption
- Key: Not rewarding q_{lr}^* for explaining long-run TFP movements

Business Cycle Co-movement



Volatility contribution at business-cycle frequency band (6-32 quarters):

Shock	Unemployment	Output	Hours Work	Investment	Consumption
Long-run	33.9	56.6	30.8	43.8	32.8
	[22.8, 46.4]	[36.1, 73.7]	[22, 41.3]	[27.4, 59.1]	[25.7, 40.2]
Short-run	46.8	42.1	39.4	41.3	23
	[34.1, 57.7]	[25.2, 63]	[28.8, 48.3]	[25.6, 57.7]	[16.1, 30.8]
Total	80.7	98.7	70.2	85.1	55.8

NOTE. 80 percent HPDI in brackets

TFP, Inflation & Interest Rates



Supply shock (TFP) $\uparrow \implies$ GDP $\uparrow \implies$ inflation $\downarrow \xrightarrow{\text{Taylor Rule}}$ nominal rates \downarrow

Demand shock $\uparrow \implies$ GDP $\uparrow \implies$ inflation $\uparrow \xrightarrow{\text{Taylor Rule}}$ nominal rates \uparrow

TFP, Inflation & Interest Rates



	TFP (6-32 Q)	TFP (> 80Q)	Inflation	Nominal Int Rates
Supply Shock (long-run)	12.7	53.1	18.2	17.6
	[5.9, 22.5]	[28.7, 71.3]	[10.4, 28.4]	[8.4, 33.1]
Demand Shock (short-run)	8.3	0.23	11.7	52.5
	[3, 17.3]	[0.03, 1.05]	[5.8, 19.8]	[39.2, 62.1]

NOTE. 80 percent HPDI in brackets

TFP, Inflation & Interest Rates



	TFP (6-32 Q)	TFP (> 80Q)	Inflation	Nominal Int Rates
Supply Shock (long-run)	12.7	53.1 (<mark>26</mark>)	18.2	17.6
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	[3, 17.3]	[0.03, 1.05]	[5.8, 19.8]	[39.2, 62.1]

NOTE. 80 percent HPDI in brackets

> The **MBC** shock is a linear combination of long-run supply and short-run demand shocks

Non-Business Cycle Shocks

Shock	Output	TFP	Investment	Consumption
Supply (Long-run Volatility)	51.8	53.7	47.9	51.4
	[26.7, 72.7]	[29.4, 71.4]	[22.4, 70.2]	[26.1, 71.9]
Demand (Long-run Volatility)	0.2	0.04	0.4	0.15
	[0.03, 1]	[0, 0.2]	[0.08, 1.9]	[0.02, 0.8]
Total (Long-run Volatility)	52	53.74	48.3	51.55
Shock	Output	TFP	Investment	Consumption
Supply (Business Cycle Volatility)	56.6	13.7	43.8	32.8
	[36.1, 73.7]	[6.5, 23.7]	[27.4, 59.1]	[25.7, 40.2]
Demand (Business Cycle Volatility)	42.1	7.4	41.3	23
	[25.2, 63]	[2.5, 15.6]	[25.6, 57.7]	[16.1, 30.8]
Total (Business Cycle Volatility)	98.7	21.1	85.1	55.8

Two business cycle shocks combined explain:

- * 98.7% of business cycle GDP volatility
- * 52% of long-run GDP volatility
- Evidence for significant fraction of long-run shocks that don't drive business cycles

Application for Policy Analysis

Application: **SVAR** identified conditional correlations $\xrightarrow{estimate}$ **DSGE** model parameters

Challenge: The trade-off between macro fit and structural accuracy for policy analysis

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Application: **SVAR** identified conditional correlations $\xrightarrow{estimate}$ **DSGE** model parameters

Challenge: The trade-off between macro fit and structural accuracy for policy analysis

Chari, Kehoe & McGrattan (2009) argue:

- New Keynesian models are not ready for quarter-to-quarter policy advice
- Due to the need for macro models to fit macro data well
- Inclusion of non-structural shocks and mechanisms to fit the macro data
- Resulting in a large number of parameters lacking consistency with microeconomic evidence
- They advocate for simpler models with fewer, well-motivated parameters based on micro evidence

Application for Policy Analysis

Application: **SVAR** identified conditional correlations $\xrightarrow{estimate}$ **DSGE** model parameters

Challenge: The trade-off between macro fit and structural accuracy for policy analysis

Argue for parameter estimation using identified business cycle shocks in two ways:

- 1. Compare the normative & policy implications of the Smets-Wouter model estimated under full-information and limited-information settings
- 2. Theoretically demonstrate that parameter estimates from full-information setting are biased in the presence of non-business cycle shocks

Smets & Wouters (2007)

Using a Bayesian likelihood approach, estimate a medium-scale DSGE model to investigate:

- * Relative empirical importance of the various frictions
- * Sources of business cycle fluctuations
- Policy analysis

Components:

- 1. Adjustment costs for investment
- 2. Capacity utilization costs
- 3. Habit persistence
- 4. Price & wage indexation and nominal rigidities
- 5. Seven structural shocks (3 supply, 4 demand)

Seven Observables: GDP, Consumption, Investment, Wages, Hours Worked, Inflation, FFR

This Paper: Estimates parameters via IRF matching

Dissecting Smets-Wouters Observables



Conclusions from empirical analysis section hold

Volatility Contributions

Shock	Output	Hours Work	Investment	Consumption
Supply (Business Cycle Volatility)	57.3	23.8	43	26.8
	[33.9, 76]	[10.6, 36.1]	[22.7, 60.3]	[18.7, 36.6]
Demand (Business Cycle Volatility)	42.5	30.7	38.4	17.9
	[23.6, 65.7]	[16, 45.6]	[20.4, 59.8]	[8.3, 27.8]
Total (Business Cycle Volatility)	99.8	54.5	81.7	44.7
Supply (Long-run Volatility)	66.5	69.9	69.11	65
	[36.9, 86.4]	[44.8, 84.4]	[40, 86.5]	[34.9, 85.3]

Shock	Inflation	FFR	Wages
Supply (Business Cycle Volatility)	22.3	15.3	19.8
	[10.3, 37.9]	[6.3, 30.9]	[10.5, 32.5]
Demand (Business Cycle Volatility)	8.3	41.2	5.4
	[3.1, 21]	[25.4, 53.6]	[2.3, 12.7]
Total (Business Cycle Volatility)	30.6	56.5	25.2
Supply (Long-run Volatility)	27.7	27.7	65.3
	[11, 56]	[13, 51.7]	[33.8, 85.2]

Supply Shock: SW Long-run TFP Shock



Demand Shock: SW Risk Premia Shock



Supply Shock: SW Long-run TFP Shock



Empirical Analysi

Normative & Policy Implications



Policy trade-offs in IRF matching estimated model.

Estimation Challenges
- I argue for downward bias in business cycle implications of DSGE models estimated using Bayesian likelihood:
 - 1. DSGE models have cross-frequency restrictions
 - 2. Presence of long-run Non-Business Cycle shocks result in downward bias

Spectral Representation of DSGE Model

Canonical representation of the DSGE model:

 $\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Psi Z_t + \Pi \zeta_t$

S_t: Endogenous Variables, **Z**_t: Exogenous Shocks, ζ_t : Expectational shocks

Assuming a state-space representation and maping to observables Y_t:

 $S_t = \Theta_1 S_{t-1} + \Theta_0 \Psi Z_t$

$$Y_t = A(L)S_t = A(L)(I - \Theta_1 L)^{-1} \Theta_0 \Psi Z_t = \mathbf{D}(L; \theta) \Theta_0(\theta) \Psi(\theta_1) Z_t$$

 θ : model parameters, θ_1 : shock standard deviations

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Model implied Spectral Density of variable k due to shock l in Y_t:

$$\mathcal{SD}(\omega, k, l; \theta, \theta_1) = \frac{1}{2\pi} \left| \mathcal{M}(\omega, y_k, l; \theta) \right|^2 \sigma_l^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, l; \theta) = \mathbf{D}^k(e^{i\omega}; \theta) \mathbf{\Theta}_0^l(\theta)$$

Log-likelihood Maximization

> Harvey 1989: The log-likelihood maximization of the model can be reduced to minimizing

$$S(\theta) = \sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2} + \sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2}$$

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Data implied volatility:

$$I(\omega_j, k) = \frac{1}{2\pi} \mathcal{D}(y_k, \omega_j, B) \sigma_B^2 + \frac{1}{2\pi} \mathcal{D}(y_k, \omega_j, INB) \sigma_{INB}^2$$

Shock Standard Deviation:

$$\tilde{\sigma}_B^2(\theta) = \frac{2\pi}{T} S(\theta)$$

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Cross-Frequency Restrictions

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Data implied volatility:

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Cross-frequency Restriction: Kolmogorov result

$$\underbrace{\sum_{k=t+1}^{T} \log \left| \mathcal{M}(\omega_{j}, y_{k}, B; \theta) \right|^{2}}_{\text{long-run}} + \underbrace{\sum_{j=1}^{t} \log \left| \mathcal{M}(\omega_{j}, y_{k}, B; \theta) \right|^{2}}_{\text{short-run}} = 0$$

Minima

Theorem:

A1: Suppose $\exists \theta^*$ s.t. $\mathcal{D}(\omega_j, y_k, B) = \mathcal{M}(\omega_j, y_k, B; \theta^*) \forall \omega_j$ where $j \in \{1, 2, \dots, T\}$

 \exists a vector of parameters (θ^*) such that model implied volatility due to ϵ_B is equal to data implied volatility of the business cycle shock at all frequencies ω_i .

Theorem:

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 \exists a vector of parameters (θ^*) such that model implied volatility due to ϵ_B is equal to data implied volatility of the business cycle shock at all frequencies ω_i .

A2: Suppose
$$\exists \theta'$$
 s.t. $\frac{\mathcal{D}(\omega_j, y_k, B) + \mathcal{D}(\omega_j, y_k, INB)\kappa}{\mathcal{D}(y_k, B, INB)} = \mathcal{M}(\omega_j, y_k, B; \theta') \forall \omega_j$ where $j \in \{1, 2, \dots, T\}$

 \exists a vector of parameters (θ') such that model implied volatility due to a ϵ_B is equal to data implied normalized volatility of both a business and non-business cycle shock

Theorem:

A1: Suppose $\exists \theta^*$ s.t. $\mathcal{D}(\omega_j, y_k, B) = \mathcal{M}(\omega_j, y_k, B; \theta^*) \forall \omega_j$ where $j \in \{1, 2, \dots, T\}$

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$$\exists \theta'$$
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 \exists a vector of parameters (θ') such that model implied volatility due to a ϵ_B is equal to data implied normalized volatility of both a business and non-business cycle shock

A3:
$$\frac{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, INB)}{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, INB)} > \frac{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, B)}{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, B)}$$

Based on the identification results.

 ϵ_{INB} : Short-run spectral density ≈ 0 and the long-run spectral density $\approx 50\%$.

 ϵ_B : Short-run spectral density is greater than it's long-run spectral density.

Theorem:

A1: Suppose $\exists \theta^*$ s.t. $\mathcal{D}(\omega_j, y_k, B) = \mathcal{M}(\omega_j, y_k, B; \theta^*) \forall \omega_j$ where $j \in \{1, 2, \dots, T\}$ A2: Suppose $\exists \theta'$ s.t. $\frac{\mathcal{D}(\omega_j, y_k, B) + \mathcal{D}(\omega_j, y_k, INB)\kappa}{\mathcal{D}(y_k, B, INB)} = \mathcal{M}(\omega_j, y_k, B; \theta') \forall \omega_j$ where $j \in \{1, 2, \dots, T\}$ A3: $\frac{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, INB)}{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, R)} > \frac{\sum_{j=t+1}^{T} \mathcal{D}(\omega_j, y_k, R)}{\sum_{j=1}^{t} \mathcal{D}(\omega_j, y_k, R)}$

Under assumptions 1, 2 & 3, the minimization of $S(\theta)$ is achieved at true parameters θ^* if and only if $\sigma_{INB}^2 = 0$.

Downward Bias for Business Cycles



- O changes such that model implied long-run volatility increases, resulting in a downward bias on short-run volatility of the model
- Argues for estimation in a limited information setting

Demand Shock: SW Risk Premia Shock



Internal vs. External Propagation





Replaced likelihood estimated σ_{RP} (0.1762) with IRF matched estimation (0.0131)

Internal vs. External Propagation



 Additionally replaced likelihood estimated investment elasticity (8.0145) with IRF matching estimated (0.0145)

Conclusion

Empirical Results:

- 1. Both long-run supply and short-run demand shocks drive business cycles
- 2. DGP also comprises long-run shocks that don't contribute to business cycles

Estimation Results:

- 1. Long-run non-business cycle shocks result in a downward bias in business cycle implications of DSGE models estimated in full-information setting
- 2. Solution: Estimation in limited information setting
 - + For instance, estimation of Smets & Wouters (2007) by IRF matching with the identified shocks.

Appendix Slides

Representation

Wold Representation:

 $Y_t = D(L)Q\varepsilon_t$

Spectral density of a variable y_i in Y_t in the frequency band $[\underline{\omega}, \overline{\omega}]$ is represented as:

$$\mathcal{D}(\mathbf{y}_{j},\underline{\omega},\bar{\omega}) = \int_{\underline{\omega}}^{\bar{\omega}} \left(\overline{D^{j}(e^{-i\omega})} D^{j}(e^{-i\omega}) \right) d\omega$$

► For instance, spectral density of GDP in business-cycle frequency band (6-32 quarters):

$$D\left(GDP,\frac{2\pi}{32},\frac{2\pi}{6}\right)$$

Business Cycle Co-movement



Volatility contribution at business-cycle frequency band (6-32 quarters):

	Unemployment	Output	hours Work	Investment	Consumption
Supply Shock	31.1	49.5	30	38.8	32.5
	[20.7, 45.8]	[30.5, 71.6]	[20.5, 40.7]	[23.4, 58]	[24.7, 39.9]
Demand Shock	49.5	49	40.7	45.8	23.4
	[35.4, 59.4]	[26.3, 67.8]	[28, 49.2]	[26.4, 61.6]	[15.7, 31.7]

NOTE. 68 percent HPDI in brackets

Check: 1

$$q_{lr}, q_{sr} \equiv \arg \max_{q_{lr}, q_{sr}} q_{lr}' D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{lr} + q_{sr}' \left(1.01 \ D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) - D\left(TFP, \frac{2\pi}{\infty}, \frac{2\pi}{80}\right)\right) q_{sr}$$

s.t. $q'_{lr}q_{lr} = 1, q'_{sr}q_{sr} = 1, q'_{lr}q_{sr} = 0$

Empirical Analys

Supply Shock IRFs



Figure IRFs

$$q_{lr}, q_{sr} \equiv \arg \max_{q_{lr}, q_{sr}} q_{lr}' D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) q_{lr} + q_{sr}' \left(1.1 \ D\left(GDP, \frac{2\pi}{32}, \frac{2\pi}{6}\right) - D\left(TFP, \frac{2\pi}{\infty}, \frac{2\pi}{80}\right)\right) q_{sr}$$

s.t. $q'_{lr}q_{lr} = 1, q'_{sr}q_{sr} = 1, q'_{lr}q_{sr} = 0$

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TFP

Supply Shock IRFs



Figure IRFs

Blanchard & Quah (1989)

Blanchard & Quah (1989):

- * A bivariate VAR analysis of real GDP and unemployment.
- * Zero long-run restriction: Only the aggregate supply shock has permanent effects on the level of real GDP.
- * The residual orthogonal shock is interpreted as an aggregate demand shock.
- * They argue aggregate demand shocks as a key driver of business cycles.



* Confounds business and non-business cycle shocks.

Barsky & Sims (2011) vs. Long-run TFP Shocks





Long-run TFP shocks from Angeletos, Collard & Dellas (2020)

Similar IRFs and business cycle volatility for macro variables.

Introduction

Overview: Model Estimation Results

In light of the evidence where we have two categories of long-run TFP shocks:

- Benchmark medium-scale DSGE models have model misspecification
 - * Similar to SVAR literature, DSGE models allow for one category of long-run TFP shocks For instance, Smets & Wouters (2007)
 - * Full-information likelihood-based estimation of these models results in biased parameters
 - * Downward bias on business cycle implications of such models.
- **Solution**: Estimation in limited information setting
 - * Estimation of Smets & Wouters (2007) by IRF matching with the identified shocks.
- Result: Wage indexation and stickiness are key for propagation mechanism relative to price & investment frictions.

Demand Shock (short-run)



NOTE. 80 percent HPDI in brackets

Supply Shock (long-run)



- Explains significant volatility of Une, Y, h, I and C at both frequency bands.
- Explains only long-run fluctuations of TFP.
- Explains significant labor productivity (Y/h) fluctuations at both frequency bands.

Output Periodogram



Figure This figure shows an estimate of the spectral density of U.S. GDP per capita filtered for periodicity above 20 quarters.

ACD (2020): MBC Shock



Figure IRFs

ACD

Normative & Policy Implications



Identification Setup

MBC shock (q₁): principal component analysis

 $\max_{q_1,q_2} q_1^{\prime} A q_1 + q_2^{\prime} A q_2$

s.t.
$$q_1'q_1 = 1, q_2'q_1 = 1, q_2'q_1 = 0$$

• This paper: extrema of sums of heterogeneous quadratic forms (A \neq B)

 $\max_{q_1,q_2} q_1' A q_1 + q_2' B q_2$

s.t.
$$q'_1q_1 = 1, q'_2q_1 = 1, q'_2q_1 = 0$$

Existence & Uniqueness: Bolla, M., Michaletzky, G., Tusnády, G., Ziermann, M. (1998)

Convergence Algorithm: Jiang & Dai (2014)

Spectral Representation of DSGE Model

Canonical representation of the DSGE model:

 $\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Psi Z_t + \Pi \zeta_t$

S_t: Endogenous Variables, **Z**_t: Exogenous Shocks, ζ_t : Expectational shocks

Assuming a state-space representation and maping to observables Y_t:

 $S_t = \Theta_1 S_{t-1} + \Theta_0 \Psi Z_t$

$$Y_t = A(L)S_t = A(L)(I - \Theta_1 L)^{-1} \Theta_0 \Psi Z_t = \mathbf{D}(L; \theta) \Theta_0(\theta) \Psi(\theta_1) Z_t$$

 θ : model parameters, θ_1 : shock standard deviations

Spectral Representation of DSGE Model

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 θ : model parameters, θ_1 : shock standard deviations

Model implied Spectral Density of variable k due to shock l in Y_t:

$$\mathcal{SD}(\omega, k, l; \theta, \theta_1) = \frac{1}{2\pi} \left| \mathcal{M}(\omega, y_k, l; \theta) \right|^2 \sigma_l^2, \quad \text{where} \quad \mathcal{M}(\omega, y_k, l; \theta) = \mathbf{D}^k(e^{i\omega}; \theta) \mathbf{\Theta}_0^l(\theta)$$

Likelihood Function of DSGE Models

Harvey 1989: The log-likelihood function of the state space model in frequency domain

$$\log L(\theta, \theta_1) = -\sum_{j=1}^{T} \left(\log \frac{1}{2\pi} \left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2 \sigma_B^2 + \frac{I(\omega_j, y_k)}{\frac{1}{2\pi} \left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2 \sigma_B^2} \right)$$

Application

Likelihood Function of DSGE Models

Harvey 1989: The log-likelihood function of the state space model in frequency domain

$$\log L(\theta, \theta_1) = -\sum_{j=1}^{T} \left(\log \frac{1}{2\pi} \left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2 \sigma_B^2 + \frac{I(\omega_j, y_k)}{\frac{1}{2\pi} \left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2 \sigma_B^2} \right)$$

• Maximising log L with respect to σ_B^2 gives:

$$\tilde{\sigma}_B^2(\theta) = \frac{2\pi}{T} \sum_{j=1}^T \frac{I(\omega_j, y_k)}{\left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2} = \frac{2\pi}{T} S(\theta)$$

Reducing the maximize log L objective to minimising

$$S(\theta) = \underbrace{\sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2}}_{\text{long-run volatility}} + \underbrace{\sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2}}_{\text{short-run volatility}}$$

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Cross-Frequency Restrictions

Simplifying objective function into long-run and short-run volatility:

$$S(\theta) = \sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2} + \sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2}$$
short-run volatility
Cross-Frequency Restrictions

Simplifying objective function into long-run and short-run volatility:

$$S(\theta) = \sum_{j=1}^{t} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2} + \sum_{j=t+1}^{T} \frac{I(\omega_j, y_k)}{\left|\mathcal{M}(\omega_j, y_k, B; \theta)\right|^2}$$
short-run volatility

Data implied volatility:

$$I(\omega_j, k) = \frac{1}{2\pi} \mathcal{D}(y_k, \omega_j, B) \sigma_B^2 + \frac{1}{2\pi} \mathcal{D}(y_k, \omega_j, INB) \sigma_{INB}^2$$

Cross-frequency Restriction: Kolmogorov result

$$\sum_{j=1}^{t} \log \left| \mathcal{M}(\omega_{j}, y_{k}, B; \theta) \right|^{2} + \sum_{j=t+1}^{T} \log \left| \mathcal{M}(\omega_{j}, y_{k}, B; \theta) \right|^{2} = 0$$

Cross-Frequency Restrictions

Suppose
$$\exists \theta^*$$
 s.t. $\mathcal{D}(y_k, \omega_j, B) = \left| \mathcal{M}(\omega_j, y_k, B; \theta^*) \right|^2 \forall \omega_j$

$$S(\theta) = \sum_{j=1}^t \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, B)\sigma_B^2 + \mathcal{D}(y_k, \omega_j, INB)\sigma_{INB}^2}{\left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2} + \sum_{j=t+1}^T \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, B)\sigma_B^2 + 0}{\left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2}$$

long-run volatility

short-run volatility

CFR

Cross-Frequency Restrictions

Suppose
$$\exists \theta^*$$
 s.t. $\mathcal{D}(y_k, \omega_j, B) = \left| \mathcal{M}(\omega_j, y_k, B; \theta^*) \right|^2 \forall \omega_j$

$$S(\theta) = \sum_{j=1}^t \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, B)\sigma_B^2 + \mathcal{D}(y_k, \omega_j, INB)\sigma_{INB}^2}{\left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2} + \sum_{j=t+1}^T \frac{1}{2\pi} \frac{\mathcal{D}(y_k, \omega_j, B)\sigma_B^2 + 0}{\left| \mathcal{M}(\omega_j, y_k, B; \theta) \right|^2}$$
short-run volatility
$$S(\theta^*) = \sum_{j=1}^t \frac{\sigma_B^2}{2\pi} + \frac{\mathcal{D}(y_k, \omega_j, INB)\sigma_{INB}^2}{\left| \mathcal{M}(\omega_j, y_k, B; \theta^*) \right|^2} + \sum_{j=t+1}^T \frac{\sigma_B^2}{2\pi} = T \frac{\sigma_B^2}{2\pi} + \sum_{j=1}^t \frac{\mathcal{D}(y_k, \omega_j, INB)\sigma_{INB}^2}{\left| \mathcal{M}(\omega_j, y_k, B; \theta^*) \right|^2}$$

• Minimizes
$$S(\theta)$$
 to $T \frac{\sigma_B^2}{2\pi}$ for true parameter (θ^*) values if $\sigma_{INB}^2 = 0$

Downward Bias for Business Cycles



- O changes such that model implied long-run volatility increases, resulting in a downward bias on short-run volatility of the model
- Argues for estimation in a limited information setting

Estimated Parameters

Parameter	Description	IRF Matching	Bayesian Likelihood
$ ho_{ga}$	Feedback TFP on exogenous spending	0.9905	0.2272
$100(\beta^{-1}-1)$	time preference rate in percent	1.7162	0.1239
α	capital share	0.178	0.2079
ψ	capacity utilization cost	0.9658	0.6723
Θ	investment adjustment cost	0.0145	8.0415
σ_c	risk aversion	1.5866	1.3295
λ	external habit degree	0.6084	0.8789
Θ	fixed cost share	1	1.4888
l_{w}	Indexation to past wages	0.8241	0.5542
ξw	Calvo parameter wages	0.86	0.8682
l_p	Indexation to past prices	Ο	0.2127
ξ_{p}	Calvo parameter prices	0.72	0.7697

Estimated Parameters

Parameter	Description	IRF Matching	Bayesian Likelihood
σ_l	Frisch elasticity	0.25	2.2934
r_{π}	Taylor rule inflation feedback	1	1.7822
$r_{\Delta y}$	Taylor rule output growth feedback	0.3835	0.0010
r_y	Taylor rule output level feedback	0.064	0.1907
ρ	interest rate persistence	0.7522	0.8283
$ ho_a$	persistence productivity shock	0.9974	0.9975
$ ho_b$	persistence risk premium shock	0.83	0.2751
$ ho_g$	persistence spending shock	0.9795	0.9810
γ	growth rate	1	1.0032
σ_a	Std. productivity shock	0.4247	0.5557
σ_b	Std. risk premium shock	0.0131	0.1762

Smets-Wouter Model

$$y_{t} = c_{y}c_{t} + i_{y}i_{t} + r^{kss}k_{y}\epsilon_{t} + \epsilon_{t}^{g}$$

$$c_{t} = \frac{\lambda/\gamma}{1 + \lambda/\gamma}c_{t-1} + \frac{1}{1 + \lambda/\gamma}E_{t}c_{t+1} + \frac{w^{ss}s^{ss}(\sigma_{c} - 1)}{c^{ss}\sigma_{c}(1 + \lambda/\gamma)}(l_{t} - E_{\ell}l_{t+1})$$

$$- \frac{1 - \lambda/\gamma}{(1 + \lambda/\gamma)\sigma_{c}}(r_{t} - E_{t}\pi_{t+1}) - \frac{1 - \lambda/\gamma}{(1 + \lambda/\gamma)\sigma_{c}}e_{t}^{b}$$
(1)

$$\begin{split} i_{l} &= \frac{1}{1 + \beta \gamma^{(1-\sigma_{c})}} i_{l-1} + \frac{\beta \beta \gamma^{(1-\sigma_{c})}}{1 + \beta \gamma^{(1-\sigma_{c})}} \mathsf{E}_{t} i_{l+1} + \frac{1}{\Theta \gamma^{2} (1 + \beta \gamma^{(1-\sigma_{c})})} q_{t} + \varepsilon_{t}^{i} \\ q_{t} &= \beta (1 - \delta) \gamma^{-\sigma_{c}} \mathsf{E}_{t} q_{t+1} - r_{t} + \mathsf{E}_{t} \pi_{t+1} + (1 - \beta (1 - \delta) \gamma^{-\sigma_{c}}) \mathsf{E}_{t} r_{t+1}^{k} - \varepsilon_{t}^{b} \\ y_{t} &= \Theta_{\rho} (\alpha k_{t}^{s} + (1 - \alpha) l_{t} + \varepsilon_{t}^{a}) \\ k_{l}^{s} &= k_{t-1} + \varepsilon_{t} \\ \varepsilon_{t} &= \frac{1 - \psi}{t} r_{t}^{k} \end{split}$$

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Empirical Analysis